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Subjective Evaluation, Ambiguity
and Relational Contracts

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Subjective evaluation, ambiguity and relational contracts

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Summary. This paper studies the implications on the combination of formal and relational contracts, of the vagueness of the subjective performance measure used by the principal in the relational contract. To formalize this vagueness, we add to the basic agency model the assumption that there are several signals available as subjective measure and that the agent has ambiguous beliefs on principal's reference signal. The greatest efficiency of the combination of contracts as compared to the simple formal contract is then no longer always significant and varies according to the agent's behavior with regard to ambiguity. Its credibility can also be compromised.

Keywords and Phrases: ambiguity, moral hazard, relational contract, subjective evaluation.

JEL Classification Numbers: D81, D82, J33.

1 Introduction

Many workers have jobs which involve multiple tasks or produce outputs that are hard to observe and assess objectively with a great degree of precision. In these cases, firms are reluctant to offer compensation based on objective measures of performance. These objective measures can cause dysfunctional behavioral responses as they reward only a subset of all things that the worker does, e.g. Holmstrom and Milgrom [16]. Their noisiness can be costly to firms as it implies higher wages to balance higher risks borne by the worker. As a result, it is often argued that firms should use rewards that are at the discretion of the superior and are then based on subjective performance measures. Promotion, salary revision, and bonuses are all common practices which are typically at the discretion of superiors.

The current theoretical literature on contracts under subjective performance measures deals with the principal-agent relationship with moral hazard, where the compensation system is based on subjective evaluations which are assumed to have the advantage of giving a more comprehensive view of the agent's performance but the disadvantage of being impossible to verify by a third party. In fact, these evaluations take into account not only quantitative but also qualitative data which tend to depend heavily on the impression of the principal. An outsider cannot verify this qualitative part of the evaluation; therefore, it is hardly legally enforceable. The literature on implicit contract, more recently called relational contract (e.g. Baker et al. [5]; like them, we also call explicit contracts "formal") that uses a repeated game analysis to provide the conditions under which these contracts are self enforcing like Bull [6], MacLeod and Malcomson [19], is then considered as useful first step to study contracts under subjective performance measures. Moreover, as these evaluations are based on more information than the publicly verifiable information, they tend to offer a more comprehensive view. The works that focus on this second characteristic study whether relational contract based on non-verifiable information can efficiently supplement formal contract based on verifiable information. They assumed that non-verifiable information gives a more accurate (e.g. Pearce and Stacchetti [24], Prendergast [25]) or a more perfect (e.g. Baker et al. [4]) performance measure than verifiable information.

However, one criticism can be made regarding these studies. This objection concerns the lack of consideration of the vagueness of the subjective evaluations. This vagueness is induced by two principal facts. First, the qualitative part of the subjective evaluations depends on the judgment of the firm, referring to its own conception of what is good or bad ; this conception is not easy to describe in clear terms to the worker. Second, the diversity and qualitative nature of the information on which the judgement can be based is such that the worker does not know exactly which information is observable and so used by the firm. Consequently, even if the firm communicates to the worker, tries to explain its conception, and reveals the type of information used, the development of its subjective performance evaluation remains to a great extent unknown to the worker.

The starting point of the present paper is therefore to introduce the vagueness feature of the subjective evaluations in the analyses of the contracts under subjective performance measure. Thus we add to the basic agency model the assumption that there are several signals available as subjective performance measure and that the agent is uncertain about the signal perceived and used by the principal. Furthermore, to emphasize the idea of vagueness, the agent is assumed not to receive enough information to form a unique prior on the set of signals. The agent’s ambiguous beliefs about the principal's reference signal are then represented by a convex set of probabilities as in the multiple priors model proposed by Gilboa and Schmeidler [11]. We consider complete or partial ignorance and adopt the Hurwicz criterion to represent the agent's preferences. This formalization allows to cover the cases in which the agent is either ambiguity averse (maximizes the minimal utility) or ambiguity seeking (maximizes the maximal utility) and also either pessimistic (underestimates the influence of his action) or optimistic (overestimates the influence of his action). The efficiency and credibility of the contract under subjective performance measures are then analyzed with taking into account these cases.

Hence our work is close to the other applications of the multiple priors model in averse selection setting by Lopez-Cuñat [18] and in price auction game by Lo [17]. It can also be compared to Ghirardato [9] and Rigotti [26], which study the consequences of the introduction of ambiguous beliefs on the standard results of the moral hazard model. Yet these papers use other theoretic uncertainty decision models and consider only verifiable performance measures. Hence Ghirardato [9] uses the Choquet expected utility model where the beliefs are represented by capacity and are not necessarily additive. Moreover, both the principal and the agent have ambiguous beliefs on the verifiable performance measures. The study of Rigotti [26] has more similarities with ours. He examines a moral hazard model where, similar to our study, only the agent has ambiguous beliefs that correspond to multiple priors. However, he applies the Bewley model and assumes that the agent is risk neutral whereas we consider also risk-averse agent. The optimal contract is then shown as having a simple two-wage structure which implies a division of all the possible outputs levels into two groups with same amount paid for all outputs belonging to the same group. MacLeod [20] obtains a similar result in a context with unverifiable subjective performance measures. His setting can seem very close to ours since he also considers that several signals can be used as subjective performance measures. However, he assumes that the principal and the agent each have their own signal that they each observe privately but whose probability distributions are unique and common knowledge. Thus, unlike us, he does not consider ambiguous beliefs. He shows that the structure of the optimal contract depends on the correlation of the two signals and therefore on the degree of agreement between the principal and the agent. Whereas he focuses on the design of the optimal contract under subjective performance measures and does not take into account the advantage of this contract with respect to the contract under objective performance measures, this last point is essential in our paper.

In fact, the first main issue of our analysis of contracts under subjective performance measure is the examination of the consequences of the introduction of the feature of vagueness of subjective evaluation into standard efficiency results. Specifically, the result is the greatest efficiency of the combination of formal and relational contracts based on objective and subjective performance measures as compared to the formal contract based on only objective measure (see e.g. Prendergast [25]). We show that once the vagueness of the subjective performance is taken into account, the greatest efficiency of the combination of formal and relational contract is no longer always significant and varies according to the agent's behavior with regard to ambiguity. When the agent is ambiguity seeking, his optimism increases the efficiency of the combination of formal and relational contracts, but when he is ambiguity averse, his pessimism can be such that it cancels out almost all the superior efficiency of this combination of contracts.

The second main issue concerns the credibility of the relational contract. We show that the vagueness of the subjective performance evaluation compromises the credibility of the relational contract in the particular case where its combination with the formal contract is the most efficient. This analysis confirms, as Chen [7] pointed out, that a certain level of trust is necessary for such contract to be most likely adopted.

The rest of this paper is organized as follows. The basic agency model and the two benchmark solutions are laid out in section 2. Section 3 presents our formalization of the vagueness of the subjective performance measure which implies ambiguous beliefs for the agent. It also presents the corresponding changes in the behavior of the agent and defines the possible types of agents. In section 4 we analyze the optimal combination of formal and relational contracts when the agent has ambiguous beliefs and compare it with benchmark solutions. Section 5 concludes the paper.

2 The Basic Agency Model

This section presents the economic environment of the basic agency model. We describe the information structure, production and preferences possibilities and the timing of events. This model is similar to Prendergast's [25]. We also present and analyze two benchmark solutions: the solution with a formal court-enforced contract and the solution when the formal contract is combined with a relational contract.

2.1 The Economic Environment

We consider a relationship between a single firm (principal) and a single worker (agent). This principal-agent relationship will be one time or repeated according to the needs of the analysis. The principal is assumed to offer a compensation contract w to the agent. The agent has an alternative employment opportunity with payoff $w_0 \geq 0$. If the agent gives his consent, then he chooses an action $a \geq 0$, which costs $C(a)$. This cost function is assumed to be quadratic for tractability, where $C(a) = \frac{c \cdot a^2}{2}$ with $c > 0$.

The agent's action is unobservable by the principal and cannot be included in the terms of the compensation contract. This asymmetric information causes a moral hazard problem. The action chosen by the agent according to his interest can be harmful to the principal. Yet this action results in a stochastic output y that accrues to the principal and corresponds to an objective measure of the agent's performance. The agent's action also affects a signal t, which constitutes a subjective performance measure. We assume that the output y and the signal t are two independent normally distributed random variables with density functions $n(a, s_y)$ and $n(a, s_t)$, respectively. As the subjective performance measure t can be considered as giving a more comprehensive view, we assume that it implies less error in measuring performance. It is then a more accurate performance measure than the objective measure y. The standard deviation of these performance measures corresponding to their measurement error are such that $s_y \geq s_t$.

The two parties are assumed to observe the realizations of the objective and subjective performance measures, $\hat{y}$ and $\hat{t}$ respectively. The objective performance measure can also be verified by a third party. It can then be the basis of a formal contract. This does not need to be the case for the subjective performance measure. It is non verifiable and can only be the subject matter of a relational contract. Hence, depending on whether the compensation contract is a formal contract or a combination of formal and relational agreements, it relates the wage w to the sole objective performance measure *y*, or to the both measures, *y* and *t*. The wage payment by the principal occurs after the realization of the objective and subjective performance measures had been completed. The formal contract or the formal part of the contract is court-enforced and will be respected with certainty. The relational agreement will be followed only if it is in the principal's interest to do it.

The principal is assumed to be risk-neutral. His payoff when the output is y and the agent's compensation is *w*, corresponds to his profit $v(y, w)$, where $v(y, w) = y - w$. The agent preferences over compensation w and action a are represented by a von Neumann-Morgenstern utility function $u(w, a)$. We consider a constant absolute risk aversion (CARA) exponential form: $u(w, a) = -\exp[-r(w - C(a))]$ where $r \geq 0$ is the risk aversion parameter, with risk neutrality when $r = 0$. As Arya et al. [2, p.11] underscore, CARA has the advantage of focusing on the first-order effect of risk aversion. It allows for multiperiod contracts with memory that cannot be mutually beneficial relative to single period arrangements when multiplicative separability across periods is assumed (see Fellingham et al. [8]). Thus the gain from long-term relationships can only be due to reputation, which helps enforcing relational agreements.

Furthermore, the exponential utility function with normally distributed performance measures implies that the optimal compensation contract gives a linear relation between wages and the performance measures (see Holmstrom and Milgrom [15]). Thus to the standard problem for establishing the optimal compensation contract, we add the ad hoc restriction that the wage function is given by: $w = s_0 + s \cdot y + b \cdot t$ where s_0 is the base salary, s is the "piece rate" on the output y and b is the "piece rate" on the signal t, with $b = 0$ when the compensation contract is only based on a formal agreement.

2.2 Benchmark One: Formal Contracts

In this subsection, we present the solution with only formal contracts (*fc*) based on the objective performance measure *y*. The relationship may be considered to be onetime. Since the agent utility function is multiplicatively separable in action and wage, the principal's problem can be decomposed into two steps (see Grossman and Hart [13]).

First, we determine the least costly way of achieving each action a. So the problem is to find for all given action a, the contract's base salary $s_0^{fc}(a)$ and the piece rate $s^{fc}(a)$ which minimize the principal's expected compensation cost subject to the two standard constraints: the agent's participation constraint (which is necessarily binding because of the multiplicative separability of the agent's utility function) and incentive compatibility constraint. From this, we deduce the minimum expected compensation cost for inducing the agent to adopt the action a, that is:

$$Ew^{fc}(a) = s_0^{fc}(a) + s^{fc}(a) \cdot a = w_0 + \frac{ca^2}{2} + \frac{ca^2}{2} rcs_y^2 \quad . \tag{1}$$

This compensation cost is composed of three parts: the same amount that the alternative employment's payoff, the cost of the action chosen and last, a risk premium, because the piece payments' part in the output *y* is uncertain and the agent can be risk averse.

Secondly, we set up the optimal action a^{fc}, which maximizes the principal's expected profit. We obtain : $a^{fc} = \frac{1}{c} \cdot s^{fc}$ with s^{fc}, the optimal formal contract piece rate equal to: $s^{fc} = \dfrac{1}{1 + rcs_y^2}$. Then, the optimal base salary, the role of which is to make the participation constraint binding, is: $s_0^{fc} = w_0 - \dfrac{1}{2c} \cdot \dfrac{1 - rcs_y^2}{\left(1 + rcs_y^2\right)^2}$ and so, we have as principal's expected profit:

$$Ev^{fc} = \frac{1}{2c} \cdot \frac{1}{1 + rcs_y^2} - w_0 \ . \qquad (2)$$

This solution can be compared with the first-best solution, attainable if the agent's choice of action is costless observable. In this situation without problem of incentive, the optimal action is $a^* = \dfrac{1}{c}$ and the agent is paying a constant wage by means of a base salary $s_0^* = w_0 + \dfrac{c \cdot a^{*2}}{2} = w_0 + \dfrac{1}{2c}$ that is equal the alternative employment's payoff plus the cost of the optimal action. This yields the profit level: $Ev^* = \dfrac{1}{2c} - w_0$.

So, departure from a first best solution where efficiency implies that the risk-neutral principal completely ensures the risk-averse agent ($r > 0$), the need of a correct incentive leads to a risk transfer from the principal to the agent. The principal must find the equilibrium between the benefits from having the correct incentives and the benefits from ensuring the agent. This equilibrium is all the more costly to achieve and the optimal action all the more inferior to the first best level since the agent is more risk-averse (r is greater) and the noisiness of the objective performance measure is higher (s_y^2 is greater). The first best action is only chosen when the agent is risk-neutral ($r = 0$) and the risk transfer for needs of incentive doesn't imply risk premium and therefore costs in efficiency.

2.3 Benchmark Two: Combination of Formal and Relational Contracts

We now consider the solution with a combination of formal and relational contracts (*mc* for mixed contract) based on the objective y and subjective t performance measures respectively and compare it with the previous case. For the moment, the particular features of the subjective performance measure are of two sorts: non verifiability and greater accuracy. These two hypotheses are those often taken into account by the recent literature (e.g. Pearce and Stacchetti [24], Prendergast [25])[1].

First, we assume that the agent believes the principal will honor the informal agreement based on the subjective performance measure t. The implications of the non-verifiability of the subjective performance measure and the credibility problem will be addressed below. Thus, for the moment, the relationship can always be supposed to be one time. The principal's optimization problem remains the same as before, except that the piece rate b on the subjective performance measure is no longer fixed at zero. We can again proceed in two steps.

The minimum expected compensation cost of the mixed contract based on the base salary $s_0^{mc}(a)$ and piece rates $s^{mc}(a)$ and $b^{mc}(a)$ on the output y and the signal t, respectively, which minimize the cost of inducing the agent to select each action a, is given by:

$$Ew^{mc}(a) = s_0^{mc}(a) + \left(s^{mc}(a) + b^{mc}(a)\right)\cdot a = w_0 + \frac{ca^2}{2} + \frac{ca^2}{2}\cdot rc s_y^2 \cdot \frac{1}{1+\frac{s_y^2}{s_t^2}}. \qquad (3)$$

The mixed contract's expected compensation cost (3) differs from the formal contract's one (1) by its smaller risk premium component. Moreover, the risk premium is smaller particularly since the variance of the signal t, s_t^2, is less than the variance of the output, s_y^2. The mixed contract is actually less costly and thus more efficient because of the Informativeness Principle (see Holmstrom [14, p.84]). The signal t independent of y is assumed, as y, to be normally distributed with a density function related to the action a. Since it provides some information about a beyond that conveyed by y, it is said to be informative. This implies that taking the signal t into account for the sharing rule makes the incentives better adjusted with less transfer of risk to the agent. The fact that the subjective performance measure is less noisy reinforces the adjustment of incentives.

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[1] Baker et al. [4] in their analysis of the substitutability of implicit and explicit contracts assume that the subjective performance measure is unverifiable and "perfect" in a particular sense which differs from the idea of accuracy. The perfection of the measure depends on its ability to cause fewer distortions in the behavior of the agent.

Moreover the optimal action a^{mc}, which maximizes principal's expected profit is: $a^{mc} = \frac{1}{c} \cdot \frac{1}{1 + rcs_y^2 \frac{s_t^2}{s_t^2 + s_y^2}} = \frac{1}{c} \cdot \left(s^{mc} + b^{mc} \right)$ with s^{mc} and b^{mc}, the optimal formal and relational contract piece rates, respectively, equal to: $s^{mc} = \frac{s_t^2}{s_t^2 + s_y^2 + rcs_t^2 s_y^2}$ and $b^{mc} = \frac{s_y^2}{s_t^2 + s_y^2 + rcs_t^2 s_y^2}$. Thus the principal's expected profit is given by:

$$Ev^{mc} = \frac{1}{2c} \cdot \frac{1}{1 + rcs_y^2 \frac{s_t^2}{s_t^2 + s_y^2}} - w_0 . \qquad (4)$$

As long as the agent is risk averse ($r > 0$), his optimal action is inferior to the first best one but superior to the optimal action with formal contract : $a^* > a^{mc} > a^{fc}$. This result confirms the greatest efficiency of the optimal mixed contract since the extent of risk transfer matters. From (2) and (4), we verify that it is uniquely in this risk averse case that the principal's expected profit is strictly greater, ($Ev^{mc} - Ev^{fc} > 0$).

However, up to this point we have assumed that the agent believes the principal will honor the informal agreement based on the subjective unverifiable performance measure t. If we drop this assumption, the optimal mixed contract developed above, called unconstrained below, must be credible to be actually preferred to the optimal formal one. We consider, therefore, that the relationship is repeated to infinity. Under these circumstances, the existence of an agent's strategy which imposes costs upon the principal when he deviates from the contract can make the contract self-enforcing and therefore credible as in Bull [6]. Let us consider the following strategy: if the principal were to renege on the informal agreement part of the contract, the agent would refuse to participate in any future mixed contracts but would accept the more attractive formal one. The principal reneges on his informal agreement when he makes a payment based on the subjective performance measure, $b^{mc} \cdot \bar{t}$, less than what he really owes, $b^{mc} \cdot \hat{t}$. The optimal formal contract corresponds to the fallback position and as we saw before, the principal's expected profit with this contract is strictly inferior to the optimal mixed contract when the agent is risk-averse ($r > 0$).

So if we assume the agent uses the above trigger strategy, the principal risks bearing costs by deviating from the mixed contract. In these circumstances, the unconstrained optimal mixed contract will remain optimal if the following credibility constraint[2] is satisfied for all realizations $\hat{t}$ of the signal t:

$$\frac{1}{r}\cdot\left[Ev^{mc}-Ev^{fc}\right]\geq b^{mc}\cdot\hat{t} \tag{5}$$

where $r\in\left[0,\infty\right)$ is the principal's discount rate.

The principal's onetime maximum gain from reneging on the informal agreement, $b^{mc}\cdot\left(\hat{t}-\bar{t}\right)$ where $\bar{t}=0$, should not exceed the present value of the difference between the mixed and the formal optimal contract's expected profit from the next period to infinity. As usual the credibility constraint is more likely to be satisfied if the principal is less impatient (r is more near 0) and his future payoffs are of more concern to him.

From this analysis, which just takes into account the greater accuracy and non-verifiability of the subjective performance measure, we conclude that as long as the agent is risk-averse ($r>0$), there exist some conditions such that the optimal mixed contract will be self-enforcing and thus credible and preferred to the formal one. We will see below that the greatest efficiency of the mixed contract may be higher or lower, or even insignificant, as soon as we consider in addition that the subjective performance measure used by the principal has the particular characteristic to be vague for the agent. In this context, the credibility of the optimal mixed contract can also be compromised.

3 The Model with Ambiguity

This section shows that the subjective performance measure used by the principal can appear vague for the agent. Then we propose a particular formalization of this vagueness which implies that the agent has ambiguous beliefs concerning the signal used as a subjective performance measure by the principal. Further, we distinguish several cases depending on whether the agent's preferences are ambiguity averse or

[2] Baker et al. [4] resort also to this trigger strategy. We can alternatively assume that if the principal were to renege on the informal agreement part of the contract, then the agent would never work for him again. But with this other trigger strategy, the credibility constraint will be easier to satisfy.

seeking and whether the signals considered by him as possible subjective performance measure are optimistic or pessimistic .

3.1 Subjective Evaluation and Ambiguous Beliefs

The subjective evaluation of the performance of a worker by a firm can be considered to be based on both quantitative and qualitative data. This is consistent with the assumptions of greater accuracy and non-verifiability of the subjective evaluation set before. This evaluation takes into account many aspects of performance and so it will be more accurate and also more difficult to verify by a third party. But the development of this evaluation also has the characteristic of being private to a great extent for the firm. The qualitative part of the evaluation depends on the judgment of the firm referring to its own conception of what is good or bad and this conception is not easy to describe in clear terms to the worker. The diversity and qualitative nature of the information on which the judgement can be based, is such that the worker does not know exactly which information is observable and therefore used by the firm. Consequently, even if the firm communicates to the worker and tries to explain its conception and the type of information used, the development of its subjective performance evaluation remains to a great extent unknown by the worker.

So we add to the basic agency model the assumption that there exist several signals which can be used as unverifiable subjective performance measures. The agent is assumed to observe the realizations of all these signals when the principal observes the realizations of only one. The agent is therefore uncertain about the signal that is observed and used by the principal. For tractability, we consider only two signals t_1 and t_2 where t_1 is the principal's reference signal. These signals are correlated with the agent's action such that they are normally distributed random variables normally distributed with the same standard deviation s_t but with a mean of a for t_1 and a mean of $[(1+q)\cdot a]$ for t_2 where $q \in (q_l, 0) \cup (0, q_h)$. The density functions of these random variables are common knowledge. The agent's action has a greater influence over one of the two signals but both are equally accurate as performance measures. We assume that $q_l = -\frac{rcs_y^2}{1+rcs_y^2}$ and $q_h = \frac{1}{2}\left(\sqrt{1+4rcs_t^2}-1\right)$.

These assumptions, as we will see later, make sure that the optimal mixed contract has strictly positive piece rates on objective and subjective performance measures.

Furthermore, to emphasize the idea of vagueness, we assume in addition that the agent has too little information to form a unique prior on the signals set $\{t_1, t_2\}$ concerning the signal used as a subjective performance measure by the principal. We are in a context said of Knightian uncertainty where the agent has ambiguous beliefs. We assume that these beliefs are represented by a set of priors as in the multiple priors model of Gilboa and Schmeidler [11][3]. Hence the agent's beliefs concerning the principal's reference signal are such that the probabilities of having t_1 and t_2 as subjective performance measure are equal to $(1-p)$ and p respectively where $p \in [p_l, p_h]$ with $0 \leq p_l < p_h \leq 1$. We obtain a closed convex set of priors on the signals set which corresponds to the space of states of nature for the agent. When $p_l = 0$ and $p_h = 1$, and the set of priors is the whole interval $[0,1]$, the agent is completely ignorant of the principal's reference signal; in the other cases, when the set of priors is a subset of the interval $[0,1]$, the ignorance of the agent is only partial.

3.2 Ambiguity Averse/Seeking and Pessimistic/Optimistic Agent

In this model, where the principal knows that several signals exist that can be used as subjective performance measures but observes the realizations of only one, the principal has no more choice than before. The principal still maximizes the expected profit where the random variables are the objective and subjective performance measures, y and $t \equiv t_1$, and nothing is changed . On the other hand, as the agent considers that the subjective performance measure t can be either t_1 or t_2, his preferences for actions represented before by the expected utility can no longer be the same. This expected utility is now not uniquely determined and depends on which probability $p \in [p_l, p_h]$ over the signals set $\{t_1, t_2\}$ concerning the principal's reference signal is used. We then assume that the agent selects the action that

[3] The multiple priors model is one of the generalizations of the subjective expected utility model aimed at explaining the Ellsberg Paradox which demonstrates that there are situations where the information possessed by a decision maker about the states of nature is too ambiguous to be represented by a unique probability measure. The other popular model is the Choquet expected utility model of Schmeidler [27].

maximizes the following function corresponding to the Hurwicz (see Arrow and Hurwicz [1]) criterion :

$$a \cdot \left[\min_{p_l \le p \le p_h} \left(p \cdot E_2 u(a) + (1-p) \cdot E_1 u(a) \right) \right] + (1-a) \cdot \left[\mathop{Max}_{p_l \le p \le p_h} \left(p \cdot E_2 u(a) + (1-p) \cdot E_1 u(a) \right) \right]$$

where $0 \le a \le 1$ and $E_i u(a)$ is the agent's expected utility from the action a when the random variables are the objective and subjective performance measure, y and $t \equiv t_i$ with $i = 1, 2$.

Hence, when $a = 1$ we are in the particular maxmin expected utility case of the multiple priors model studied by Gilboa and Schmeidler [11]. The agent evaluates each action by computing the minimum expected utility over the probability measures of his priors set and the most emphasis is given to the worst event. When $a = 0$ maxmax replaces maxmin and the most emphasis is given to the best event. In the first case, the agent is said ambiguity averse, while the agent is ambiguity seeking in the second (see Ghirardato and Marinacci [10, p.603])[4].

Furthermore, when $0 < q < q_h$, the signals t_1 and t_2 considered by the agent as available subjective performance measures have a mean greater than or equal to the mean of the subjective performance measure $t \equiv t_1$ considered by the principal. Then the agent tends to overestimate the influence of his action on his performance. On the other hand, when $q_l < q < 0$ the situation is inverse and underestimation occurs. In the first case, the agent will be said optimistic and pessimistic in the second.

Consequently, when we only consider the two extreme cases where the agent is either ambiguity averse or seeking, we distinguish four situations with different types of agent that can be submit by the following summary table 1:

Table 1. Agent's Types

	$a = 1$	$a = 0$
$0 < q < q_h$	Ambiguity Aversion and Optimism	Ambiguity Seeking and Optimism
$q_l < q < 0$	Ambiguity Aversion and Pessimism	Ambiguity Seeking and Pessimism

[4] Other denominations can be found as uncertainty averse and seeking in Gilboa and Schmeidler [12, p.42], pessimistic and optimistic in Lopez-Cuñat [18, p.380].

These four different types of agent will be taken into account in the design of the optimal mixed contract. From this point forward both parties are assumed to know the type of the agent.

4 Combination of Formal and Relational Contracts in a Setting with Ambiguity

In this setting with ambiguity, we first consider that the agent believes that the principal will not renege on his informal agreement and we establish the different optimal compensation mixed contracts in accordance with the four types of agents. We assume that the agent is always risk averse ($r>0$). We verify, therefore, that the optimal mixed contract remains more efficient than the optimal formal contract regardless of the agent's type. Yet, the comparison with the case without ambiguous beliefs on the part of the agent shows that the superior efficiency of the mixed contract is more or less significant depending on whether the agent is ambiguity seeking or averse. Moreover, the consideration of the fact that the principal may renege on his informal agreement shows that the credibility of the optimal mixed contract can never be guaranteed in the greatest efficiency case where the agent is ambiguity seeking when the signals considered as possible subjective measures are sufficiently different.

4.1 The Optimal Compensation Contracts

Let us first consider that the agent trusts the principal and thinks that he will not renege on his informal agreement. The principal-agent relationship is then considered to be onetime. Furthermore, we assumed that the agent's type is common knowledge. The compensation mixed contract offered by the principal to the agent takes into account his type. We also assumed that the principal has private information. This information concerns the signal used as subjective performance measure. The structure of the mixed contract proposed may reveal some of what he knows and may allow the agent to revise his prior beliefs regarding the principal's reference signal. Yet contrary to the literature on signaling models like Maskin and Tirole [21] and

[22][5], we assume that the agent is not able to use the information conveyed by the contract proposal. Therefore, the optimal compensation mixed contract when the agent has ambiguous beliefs on the signal used as subjective performance measure (*amc* for mixed contract with ambiguity) is either the solution of the principal's optimization problem that follows if the agent is ambiguity seeking:

$$\max_{a,so,s,b} \quad a - (s_0 + s \cdot a + b \cdot a)$$

subject to the participat ion constraint

$$\max_{p_l \le p \le p_h} p \cdot E_2 u(a) + (1-p) \cdot E_1 u(a) \ge -\exp[-rw_0]$$

and the incentive compatibility constraint

$$a \in \arg\max_{\tilde{a} \ge 0} \left[\max_{p_l \le p \le p_h} p.E_2 u(\tilde{a}) + (1\text{-}p) \cdot E_1 u(\tilde{a}) \right]$$

or of the following problem if the agent is ambiguity averse:

$$\max_{a,so,s,b} \quad a - (s_0 + s \cdot a + b \cdot a)$$

subject to the participat ion constraint

$$\min_{p_l \le p \le p_h} p \cdot E_2 u(a) + (1-p) \cdot E_1 u(a) \ge -\exp[-rw_0]$$

and the incentive compatibility constraint

$$a \in \arg\max_{\tilde{a} \ge 0} \left[\min_{p_l \le p \le p_h} p.E_2 u(\tilde{a}) + (1\text{-}p) \cdot E_1 u(\tilde{a}) \right]$$

For these two cases we have:

$$E_1 u(a) = -\exp\left[-r\left(s_0 - \frac{ca^2}{2}\right)\right] \cdot \exp\left[-\frac{rs}{2}\left(2a - rs s_y^2\right)\right] \cdot \exp\left[-\frac{rb}{2}\left(2a - rbs_t^2\right)\right]$$

and

$$E_2 u(a) = -\exp\left[-r\left(s_0 - \frac{ca^2}{2}\right)\right] \cdot \exp\left[-\frac{rs}{2}\left(2a - rs s_y^2\right)\right] \cdot \exp\left[-\frac{rb}{2}\left(2a(1+q) - rbs_t^2\right)\right]$$

[5] Maskin and Tirole analyze the efficiency of the principal-agent relationship with an informed principal and consider two cases: the common values [22] and the private values [21] depending on whether the principal's private information directly affects the agent's payoff. Furthermore, they assume that actions are observable and verifiable, thereby ruling out moral hazard. In our case, we have private information of the common values form but the moral hazard problem remains a main problem. In fact, we would like to focus on the impact of the agent's ambiguous beliefs about the principal's reference signal on the efficiency of the incentives and we have chosen to rule out any signaling at the stage of the contract proposal. It does not prevent us from considering signaling at the stage of the contract execution, as we will do below.

with $q_l < q < 0$ when the agent is pessimistic or $0 < q < q_h$ when he is optimistic.

Note $s_0^{amc}(a)$, the base salary and, $s^{amc}(a)$ and $b^{amc}(a)$, the piece rates on the output *y* and the principal's reference signal $t \equiv t_1$, respectively, which minimize the cost of inducing the agent to select each action *a*. This cost is then characterized by the following Lemma. All proofs are deferred to the appendix.

Lemma 1. In a setting with ambiguity, the mixed contract's minimum expected compensation cost can be defined by one unique general formulation as follows:

$$\begin{aligned} Ew^{amc}(a) &= s_0^{amc}(a) + \left(s^{amc}(a) + b^{amc}(a)\right) \cdot a \\ &= w_0 + \frac{ca^2}{2} \\ &+ \frac{ca^2}{2} \cdot rcs_y^2 \cdot \frac{s_t^2}{s_t^2 + s_y^2(1+\tilde{q})^2} + \frac{a^2 \cdot \tilde{q}^2}{2r\left(s_t^2 + s_y^2(1+\tilde{q})^2\right)} \\ &- a^2 \cdot \tilde{q} \cdot \left[\frac{rcs_y^2(1+\tilde{q}) + \tilde{q}}{r\left(s_t^2 + s_y^2(1+\tilde{q})^2\right)}\right] \end{aligned} \tag{6}$$

where (i) $\tilde{q} = 0$ if $p_l = 0$ and the agent is either ambiguity seeking and pessimistic or ambiguity averse and optimistic, and (ii) $\tilde{q} = q$ if $p_h = 1$ and the agent is either ambiguity seeking and optimistic or ambiguity averse and pessimistic.

Therefore, the optimal action a^{amc} and its associated principal's maximum expected profit Ev^{amc} can also be defined by one unique general formulation, with the same conditions (i) and (ii). We obtain : $a^{amc} = \frac{1}{c} \cdot \left[s^{amc} + (1+\tilde{q}) \cdot b^{amc}\right]$ with

$s^{amc} = \dfrac{rcs_t^2 - \tilde{q} \cdot (1+\tilde{q})}{rc\left[s_t^2 + s_y^2\left(1-\tilde{q}^2\right) + rcs_y^2 s_t^2\right] - \tilde{q}^2}$ the optimal formal contract 's piece rate

and $b^{amc} = \dfrac{rc(1+\tilde{q})s_y^2 + \tilde{q}}{rc\left[s_t^2 + s_y^2\left(1-\tilde{q}^2\right) + rcs_y^2 s_t^2\right] - \tilde{q}^2}$, the optimal relational contract's piece

rate. Then we have $Ev^{amc} = \dfrac{1}{2c} \cdot \dfrac{rcs_t^2 + rc(1+\tilde{q})^2 s_y^2}{rc\left[s_t^2 + s_y^2\left(1-\tilde{q}^2\right) + rcs_y^2 s_t^2\right] - \tilde{q}^2} - w_0$ as principal's

maximum expected profit.

Under partial ignorance, the assumptions on the limit values of the priors ($p_l = 0$ or $p_h = 1$) imply that the set of priors contains the belief for which the principal's reference signal is either the more sensible to the agent's action with probability one when the agent is ambiguity seeking or the less sensible to the agent's action with probability one when the agent is ambiguity averse. Under complete ignorance, these assumptions are both satisfied and the conditions (i) and (ii) can be simplified and merely specify the agent's types. The minimum expected compensation cost, the optimal action and the principal's maximum expected profit differ then solely according to the agent's type.

4.2 Impact of the Ambiguity on the Contracts' Efficiency

From the only consideration of the minimum expected compensation cost which allow to achieve each action a and so from Lemma 1 , we can show that in a setting with ambiguity the optimal mixed contract remains more efficient than the optimal formal contract but that this difference in efficiency may be much less significant. We have the first following proposition:

Proposition 1. Under complete or partial ignorance where $p_l = 0$, the vagueness of the subjective performance measure used by the principal in the relational part of the mixed contract has no impact on the efficiency of this contract, if the agent is either ambiguity seeking and pessimistic or ambiguity averse and optimistic.

Hence, in this case the optimal mixed contract, with or without ambiguity, is more efficient in the same way than the formal contract.

The second proposition of this section is the following:

Proposition 2. Under complete or partial ignorance where $p_h = 1$, the vagueness of the subjective performance measure used by the principal in the relational part of the mixed contract has a positive impact on the efficiency of this contract, if the agent is ambiguity seeking and optimistic. But its impact is negative and may even cancel out almost all of the superior efficiency of the optimal mixed contract with respect to the optimal formal contract if the agent is ambiguity averse and pessimistic.

In fact, the mixed contract's minimum expected compensation cost (6) with $\tilde{q} = q$ is composed of different parts. As previously for the mixed contract without ambiguity (3) and the formal one (1), we get the same amount that the alternative employment's payoff (w_0), the cost of the action chosen ($\frac{ca^2}{2}$) and a risk premium which corresponds here to the sum of the third and fourth terms. But from these three first parts a last term is subtracted, which can be defined as the saving on incentives (this saving is in fact cost overrun when this last term is negative) due to the fact that the agent tends to overestimate (or underestimate) the influence of his action on his performance. As we have $b^{amc}(a) = a \cdot \left[\frac{rcs_y^2(1+q)+q}{r\left(s_t^2 + s_y^2(1+q)\right)} \right] > 0$ for all $q \in (q_l, 0) \cup (0, q_h)$, we can verify that the last term is equal to $b^{amc}(a) \cdot q \cdot a$, which corresponds to the possible agent's overestimation ($q \in (0, q_h)$) or underestimation ($q \in (q_l, 0)$) of the expected payoff when the agent has ambiguous beliefs. It is the combined variations of the risk premium and the saving on incentives that determine the impact of the ambiguity on the efficiency of the mixed contract. The risk aversion of the agent is then no longer sufficient to guarantee a significant greatest efficiency of the mixed contract and the other characteristics of the agent like his optimism/pessimism and his ambiguity aversion/seeking matter as well.

4.3 Incompatibility between Greatest Efficiency and Credibility

Even if the difference in efficiency between the optimal mixed contract and the optimal formal contract is not always significant when the agent has ambiguous beliefs regarding the principal's reference signal used as a subjective performance measure, the mixed contract remains more efficient. However, up to this point we have assumed that the agent trusts the principal. Therefore, he believes that the principal will honor the informal agreement based on the subjective performance measure assumed to be non-verifiable by a third party. If we drop this assumption, the unconstrained optimal mixed contract must be credible in order to be preferred to the optimal formal contract. Traditionally, as we have seen before in the case without ambiguity, we turn to a repeated game analysis. In this case, a contract with informal unverifiable agreements will be credible if there exists an agent's strategy which imposes costs upon the principal when he deviates from these agreements. This trigger strategy makes the contract self-enforcing for the principal.

Yet in this setting with ambiguity, the subjective performance measure is assumed to be vague for the agent. This vagueness implies that the agent considers several signals that the principal could use as subjective performance measure and does not know which signal is used by the principal. The agent is assumed to observe the realizations of all the signals, whereas the principal observes only the realizations of his reference signal. Hence this vagueness implies several important changes. First, the deviation on the part of the principal is only obvious when the reference value $\bar{t}$ for the subjective piece payment is distinct from the realizations of both signals, $\hat{t}_1$ and $\hat{t}_2$. It is only when the realizations of the signals $\hat{t}_1$ and $\hat{t}_2$ are both equal to the reference value $\bar{t}$ that the respect of the agreement is sure. Second, when the realizations of both signals, $\hat{t}_1$ and $\hat{t}_2$, are distinct and the reference value $\bar{t}$ equals one of them, the agent may take this as a disclosure of the reference signal of the principal but can be misled. Thirdly, the principal knows, of course, when he deviates or not, but neither knows if his behavior is clearly identified nor if he reveals his reference signal or mislead the agent.

The situation is therefore more complex. Let us consider that the relationship is repeated to infinity. Then the agent's trigger strategy can only be as follows. If the agent is sure that the principal reneged on the informal agreement part of the contract, the agent will refuse to participate in any future mixed contracts but would accept the more attractive formal one. The optimal formal contract corresponds then to the fallback position. On the other hand, if he has doubts about whether the principal reneged and the payoff seems induce a particular reference signal, the agent takes it into account in future periods and all future disrespect of this supposed reference signal will be punished as mentioned above. The repetition of the relationship may then have the effects of both an enforcement mechanism and a signaling mechanism. Moreover, as the principal does not know how his payoff will be interpreted by the agent, this trigger strategy implies for him a stochastic punishment. Furthermore, we assumed before that the agent is not able to use the information conveyed by the contract proposal. If so, it may imply that the agent does not know how the principal designs his contract proposal and is not able to anticipate changes in the contract proposal for coming periods. In these circumstances, the agent does not see things exactly the same way as the principal does. Hence the mixed contract will be credible if it seems self-enforcing for the principal in accordance with the point of view of the agent and his own conception of the repeated game. His conception is assumed to be such that the only future alternatives are: the same mixed contract again ($w^{amc} = s_0^{amc} + s^{amc} \cdot y + b^{amc} \cdot t$), the optimal formal contract ($w^{fc} = s_0^{fc} + s^{fc} \cdot y$) or the other employment opportunity (w_0).

Therefore the agent's trigger strategy implies the following credibility constraints:

$$\left[\hat{y} - \left(s_0^{amc} + s^{amc} \cdot \hat{y} + b^{amc} \cdot \hat{t}_i\right)\right] + \sum_{n=0}^{\infty} prob\left(t_j = \hat{t}_i\right) \cdot prob\left(t_j = t_i\right)^n \cdot \frac{1}{(1+r)^{n+1}} \cdot Ev^{amc(i)}$$

$$+ \left[1 - prob\left(t_j = \hat{t}_i\right)\right] \cdot \frac{1}{r} \cdot Ev^{(i)} + \sum_{n=0}^{\infty} prob\left(t_j = \hat{t}_i\right) \cdot prob\left(t_j = t_i\right)^n \cdot \left[1 - prob\left(t_j = t_i\right)\right] \cdot \frac{1}{r} \cdot Ev^{(i)}$$

$$\geq$$

$$\left[\hat{y} - \left(s_0^{amc} + s^{amc} \cdot \hat{y} + b^{amc} \cdot \bar{t}\right)\right] + \sum_{n=0}^{\infty} prob\left(t_j = \bar{t}\right) \cdot prob\left(t_j = t_i\right)^n \cdot \frac{1}{(1+r)^{n+1}} \cdot Ev^{(j)}$$

$$+ \left[1 - prob\left(t_j = \bar{t}\right)\right] \cdot \frac{1}{r} \cdot Ev^{fc} + \sum_{n=0}^{\infty} prob\left(t_j = \bar{t}\right) \cdot prob\left(t_j = t_i\right)^n \cdot \left[1 - prob\left(t_j = t_i\right)\right] \cdot Ev^{fc}$$

for all $i, j = 1, 2$ with $j \neq i$, (7)

and where

$$Ev^{amc(i)} = a^{amc} - \left(s_0^{amc} + s^{amc} \cdot a^{amc} + b^{amc} \cdot E\left(t_i\right)\right) \text{ for all } i = 1, 2$$

$$Ev^{(1)} = \begin{cases} a^{(1)} - \left(s_0^{amc} + s^{amc} \cdot a^{(1)} + b^{amc} \cdot a^{(1)}\right) \\ \text{where } a^{(1)} \in \arg\max_a E\left[-\exp\left(-r \cdot \left(s_0^{amc} + s^{amc} \cdot y + b^{amc} \cdot t_1 - \frac{ca^2}{2}\right)\right)\right] \\ if \ -\exp\left[-r\left(s_0^{amc} - \frac{ca^{(1)2}}{2}\right)\right] \cdot \exp\left[-\frac{rs^{amc}}{2}\left(2a^{(1)} - rs^{amc}s_y^2\right)\right] \\ \quad \cdot \exp\left[-\frac{rb^{amc}}{2}\left(2a^{(1)} - rb^{amc}s_t^2\right)\right] \geq -\exp\left(-rw_0\right) \\ Ev^{fc} \quad \text{otherwise} \end{cases}$$

$$Ev^{(2)} = \begin{cases} a^{(2)} - \left(s_0^{amc} + s^{amc} \cdot a^{(2)} + b^{amc} \cdot (1+q) \cdot a^{(2)}\right) \\ \text{where } a^{(2)} \in \arg\max_a E\left[-\exp\left(-r \cdot \left(s_0^{amc} + s^{amc} \cdot y + b^{amc} \cdot t_2 - \frac{ca^2}{2}\right)\right)\right] \\ if \ -\exp\left[-r\left(s_0^{amc} - \frac{ca^{(2)2}}{2}\right)\right] \cdot \exp\left[-\frac{rs^{amc}}{2}\left(2a^{(2)} - rs^{amc}s_y^2\right)\right] \\ \quad \cdot \exp\left[-\frac{rb^{amc}}{2}\left(2a^{(2)}(1+q) - rb^{amc}s_t^2\right)\right] \geq -\exp\left(-rw_0\right) \\ Ev^{fc} \quad \text{otherwise} \end{cases}$$

As the agent does not know the principal's reference signal at the proposal stage, the credibility of the mixed contract must be guaranteed, regardless of it and we have two credibility constraints. These two credibility constraints must be satisfied by all realizations $\hat{t}_i$ of the reference signal t_i.

The left-hand side of the inequality (7) is the present discounted value according to the agent of profit earned by the principal from the current period until infinity if his reference signal is t_i and he pays what he owes. It takes into account that the beliefs of the principal concerning the identification of his honest behavior are $prob\left(t_j = \hat{t}_i\right)$ at the current period and $prob\left(t_j = t_i\right)$ at the subsequent periods and that at all periods, the principal may reveal his reference signal with the complementary probabilities. As long as honest behavior is identified without reference signal be revealed, the agent still accepts the mixed contract. Yet, the principal's profit is not the same from the point of view of the agent and it depends of the supposed reference signal; we then have $Ev^{amc(i)}$ for all $i = 1, 2$. The disclosure is supposed to yield the profit level $Ev^{(i)}$ for the rest of the periods. The agent takes into account the disclosed signal as principal's reference signal in his optimization program where the mixed contract is unchanged and so, either the participation constraint is satisfied or not. In this last case where the mixed contract is no longer individually rational, the agent turns then to the formal contract.

The right-hand side is the present discounted value of profit earned, again according to the agent, by the principal from the current period until infinity if he breaks the promises and refers to a value $\bar{t} < \hat{t}_i$ for the payoff based on the subjective performance measure at the current period. If the principal is found to be dishonest, its promises will not be subsequently credible; it will thus earn the profit of the formal contract Ev^{fc} from that period onward. The principal's beliefs concerning the fact that he may escape being caught are $prob(\bar{t} = t_j)$ at the current period and $prob(t_i = t_j)$ at the subsequent period. As long as the principal's deviation is not obvious, the agent is mislead about the reference signal. That yields in accordance to him the principal's profit level $Ev^{(j)}$ which supposes that the agent chooses the optimal action given this signal with the same mixed contract or turn to the formal contract if the mixed contract satisfies no longer the participation contract.

Hence, the unconstrained optimal mixed contract will exist and remain optimal if the credibility constraints (7) are satisfied. We establish the following proposition:

Proposition 3. In a setting where the subjective performance measure is vague for the agent and the signals considered as possible subjective performance measures are sufficiently different, the credibility of the unconstrained optimal mixed contract can not always be guaranteed by the low impatience of the principal as usual. This situation is possible only when the agent is ambiguity averse. When the agent is ambiguity seeking the contract is never credible.

In fact, when the agent is ambiguity seeking and puts more emphasis on the best event, the disclosure of the signal can reveal an overestimation of the mixed contract's payoff whereas it cannot be so when the agent is ambiguity averse and puts more emphasis on the worst event. Hence, it is in the case where the efficiency superiority of the combination of formal and relational contracts is the greatest that credibility can not be guaranteed.

5 Conclusion

The present paper has explored the implications on the efficiency of the combination of formal and relational contracts, of the fact that the subjective performance measure used by the principal, non verifiable and more accurate than objective one, has the additional characteristic of being vague for the agent. To formalize this vagueness, we added to the basic agency model the assumption that there exist several signals which can be used as subjective performance measures by the principal and that the agent has ambiguous beliefs concerning the principal's reference signal. Specifically, we let the agent's beliefs be represented by a convex set of probabilities on the signals set. The agent's behavior is then described by a general function that covers the cases in which the agent is either ambiguity averse (maximizes the minimal utility) or ambiguity seeking (maximizes the maximal utility) and also either pessimistic (underestimates the influence of his action) or optimistic (overestimates the influence of his action). Therefore, our formulation implies that the efficiency of the optimal combination of formal and relational contracts varies according to the agent's type.

The main result of our analysis is that once the vagueness of the subjective performance measure is taken into account, the number of situations where the combination of formal and relational contracts are preferred to a formal contract is reduced in a significant way. The greatest efficiency of the combination of formal and relational contracts as compared to the formal contract can be greater significant when the agent is ambiguity seeking but the relational agreement will not be credible. When the agent is ambiguity averse and there is some chance that the relational agreement will be credible, the pessimism of the agent can be such that the combination of formal and relational contracts will not be much more efficient than the simple formal one. Thus our analysis shows that the lack of trust makes these combinations of contracts less likely to be adopted. This confirms the intuition of Baker, Jensen and Murphy [3] who pointed out that the lack of trust may lead to evasion of compensation systems based on subjective performance measures. Our analysis implies moreover that the lack of optimism also matters. This might explain the general reluctance of managers to give poor performance evaluations to employees, as noted by Medoff and Abraham [23] and called "leniency bias" (see e.g. Prendergast [25, p.30]). This aversion to giving subordinates poor evaluations may be induced by the needs of employees' optimism for efficiency of compensation system based on subjective performance measure.

Appendix

Proof of Lemma 1

As $(E_2u(a)-E_1u(a))\cdot(E_2u(a')-E_1u(a'))\geq 0$ for every action a and a', the agent's actions are comonotonic and the minimizing or maximizing probability will be the same for all action a. The principal's problem can then be simplified. Under complete ignorance ($p_l=0$, $p_h=1$), the incentive compatibility constraint and the participation constraint will be replaced by $a\in\arg\max_{\tilde{a}\geq 0}\ [\max(E_2u(\tilde{a}),E_1u(\tilde{a}))]$ and $\max(E_2u(a),E_1u(a))\geq -\exp[-rw_0]$, respectively, if the agent is ambiguity seeking. They will be replaced by $a\in\arg\max_{\tilde{a}\geq 0}\ [\min(E_2u(\tilde{a}),E_1u(\tilde{a}))]$ and $\min(E_2u(a),E_1u(a))\geq -\exp[-rw_0]$, respectively, if the agent is ambiguity averse. Under partial ignorance with either $p_l=0$ or $p_h=1$, similar simplifications can be made. The rest of the proof is immediate once the following facts are noted: $E_2u(a)<E_1u(a)\ \ if\ \ q_l<q<0$ and $E_1u(a)<E_2u(a)\ \ if\ \ 0<q<q_h$. QED

Proof of proposition 1

Just notice that from Lemma 1 we have $\tilde{q}=0$ which imply (6) is equal to (3). QED

Proof of proposition 2

From Lemma 1, $\tilde{q}=q$ and so the impact of the ambiguity is function of q and may be deduced from the analysis of the variations of the minimum expected compensation cost (6) with respect to $q \in (q_l, 0) \cup (0, q_h)$ with $q_l = -\frac{rcs_y^2}{1 + rcs_y^2}$ and $q_h = \frac{1}{2}\left(\sqrt{1 + 4rcs_t^2} - 1\right)$, as assumed before.

The derivative of this expected cost (6) with $\tilde{q} = q$ is in fact such that

$$\frac{\partial Ew^{amc}(a)}{\partial q} = -a^2 \cdot \frac{\left(rcs_y^2(1+q)+q\right)\cdot\left(s_t^2 + s_y^2(1+q) + rcs_y^2 s_t^2\right)}{r \cdot \left(s_t^2 + s_y^2(1+q)^2\right)} < 0 \quad \text{for all}$$

$q \in (q_l, 0) \cup (0, q_h)$. Furthermore, its limit values are at $q = q_l$, equal to (1) and at $q = 0$, equal to (3). The proposition 2 is then straightforward to check. QED

Proof of proposition 3

The credibility constraints (7) are equivalent to:

$$\left[\hat{y} - \left(s_0^{amc} + s^{amc}\cdot\hat{y} + b^{amc}\cdot\hat{t}_i\right)\right] + prob\left(t_j = \hat{t}_i\right)\cdot\frac{1}{1 + r - prob\left(t_j = t_i\right)}\cdot Ev^{amc(i)} + \frac{1}{r}\cdot Ev^{(i)} \geq$$

$$\left[\hat{y} - \left(s_0^{amc} + s^{amc}\cdot\hat{y} + b^{amc}\cdot\bar{t}\right)\right] + \frac{1}{r}\cdot Ev^{fc} + prob\left(t_j = \bar{t}\right)\cdot\frac{1}{1 + r - prob\left(t_j = t_i\right)}\cdot Ev^{(j)}$$

for all $i, j = 1, 2$ with $j \neq i$,

which yields

$$b\left(\hat{t}_i - \bar{t}\right) \leq \frac{1}{r}\cdot\left(Ev^{(i)} - Ev^{fc}\right) + \frac{1}{1 + r - prob\left(t_j = t_i\right)}\cdot\left[prob\left(t_j = \hat{t}_i\right)\cdot Ev^{amc} - prob\left(t_j = \bar{t}\right)\cdot Ev^{(j)}\right]$$

for all $i, j = 1, 2$ with $j \neq i$. (8)

Then if the signals are different enough, the probability that their realizations have same value is distinct from unity ($prob(t_1 = t_2) << 1$) and it is necessary that the difference in expected profit $\left(Ev^{(i)} - Ev^{fc}\right)$ will be strictly positive in order to satisfy the credibility constraints when the principal is less impatient (r is more near 0) as usual. Afterwards as the mixed contracts vary in accordance of the agent's type, the analysis must be done case by case. So, when the agent is ambiguity seeking, one may verify that $Ev^{(1)} = Ev^{fc}$ if the agent is optimistic and $Ev^{(2)} = Ev^{fc}$ if he is pessimistic. In these cases the disclosure of the signal reveals an overestimation of the mixed contract's payoff such that the action does not allow the restoration of individual rationality and the agent then turns to the formal contract. This implies that there is always one of the two credibility constraints which is not satisfied when the agent is ambiguity seeking. On the other hand, when the agent is ambiguity averse, the situation can never become worse after a disclosure of the signal, regardless of its nature. In these circumstances, the mixed contract remains individual rational and one verifies that $Ev^{(i)} \geq Ev^{fc}$ for all $i = 1, 2$; the credibility constraints will be both satisfied if $Ev^{(i)} > Ev^{fc}$ for all $i = 1, 2$. QED

References

1. Arrow, K. J., Hurwicz L.: An optimality criterion for decision-making under ignorance. In: Carter C., Ford J. (eds.) Uncertainty and expectations in economics, pp. 1-11. Oxford: Basil Blackwell & Mott, Ltd. 1972
2. Arya, A., Fellingham, J.C., Glover, J.: Teams, repeated tasks, and implicit incentives. Journal of Accounting and Economics **23**, 7-30 (1997)
3. Baker, G., Jensen, M.C., Murphy, K.J.: Compensation and incentives: Practice vs. theory. Journal of Finance **43**, 593-616 (1988)
4. Baker, G., Jensen, M.C., Murphy, K.J.: Subjective performance measures in optimal incentive contracts. Quarterly Journal of Economics **109**, 1125-1156 (1994)
5. Baker, G., Gibbons, R., Murphy, K.J.: Relational contracts and the theory of the firm. Quarterly Journal of Economics **117**,39-84 (2002)
6. Bull, C.: The existence of self-enforcing implicit contracts. Quarterly Journal of Economics **102**, 147-159 (1987)
7. Chen, Y.: Promises, trust, and contracts. Journal of Law, Economics & Organization **16**, 209-232 (2000)
8. Fellingham, J. C., Newman, P.D., Suh, Y.S.: Contracts without memory in multiperiod agency models. Journal of Economic Theory **37**, 340-355 (1985)
9. Ghirardato, P.: Agency theory with non-additive uncertainty. Discussion paper, California Institute of Technology, Pasadena (1994)
10. Ghirardato, P., Marinacci, M.: Range convexity and ambiguity averse preferences. Economic Theory **17**, 599-617 (2001)
11. Gilboa, I., Schmeidler, D.: Maxmin expected utility with non-unique prior. Journal of Mathematical Economics **18**, 141-153 (1989)
12. Gilboa, I., Schmeidler D.: Updating ambiguous beliefs. Journal of Economic Theory **59**, 33-49 (1993)

13. Grossman, S.J., Hart, O.D.: An analysis of the principal-agent problem. Econometrica **51**, 7-45 (1983)
14. Holmstrom, B.: Moral hazard and observability. Bell Journal of Economics **10**, 74-91 (1979)
15. Holmstrom, B., Milgrom, P.: Aggregation and linearity in the provision of intertemporal incentives. Econometrica **55**, 303-328 (1987)
16. Holmstrom, B., Milgrom, P.: Multitask principal-agent analyses: Incentive contracts, asset ownership, and job design. Journal of Law, Economics & Organization **7**, 24-52 (1991)
17. Chung, L.K.: Sealed bid auctions with uncertainty averse bidders. Economic Theory **12**, 1-20 (1998)
18. Lopez-Cuñat, J.M.: Adverse selection under ignorance. Economic Theory **16**, 379-399 (2000)
19. MacLeod, B.W., Malcomson, J.M.: Implicit contracts, incentive compatibility, and involuntary unemployment. Econometrica **57**, 447-480 (1989)
20. MacLeod, B.W.: Optimal contracting with subjective evaluation. Discussion paper, University of Southern California, Los Angeles (2002)
21. Maskin, E., Tirole, J.: The principal-agent relationship with an informed principal: The case of private values. Econometrica **58**, 379-409 (1990)
22. Maskin, E., Tirole J.: The principal-agent relationship with an informed principal, II: The case of common values. Econometrica **60**, 1-42 (1992)
23. Medoff, J.L., Abraham, K.G.: Experience, performance, and earnings. Quarterly Journal of Economics **95**, 703-736 (1980)
24. Pearce, D.G., Stacchetti, E.: The interaction of implicit and explicit contracts in repeated agency. Games and Economic Behavior **23**, 75-96 (1998)
25. Prendergast, C.: The provision of incentives in firms. Journal of Economic Literature **37**, 7-63 (1999)
26. Rigotti, L.: Imprecise beliefs in a principal agent model. Discussion paper, U.C. Berkeley (2001)
27. Schmeidler, D.: Subjective probability and expected utility without additivity. Econometrica **57**, 571-587 (1989)