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*Majority voting with stochastic preferences :
The whims of a committee are smaller than the whims
of its members*

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Abstract : We study the volatility of the policy chosen by a committee whose members have volatile preferences. It is smaller than if it was chosen by a single member, smaller the larger the size of the committee, and smaller the volatility of members' preferences.

Keywords : committee, majority voting, uncertainty, volatility.
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1. Introduction

"The mutability of the laws is an evil inherent in democratic government". Tocqueville (1835, chapter 15)'s pessimistic assertion rests on the presumption that a democratic country's laws are subject to the "daily passion" of its constituency. In nowadays words, changes in voters' preferences are a source of volatility of chosen policies.

Strikingly, as Schofield (2002) stresses, most social choice theory usually assumes that voters have well-defined and stable preferences, or that their ideal policies are given. One may however think of numerous phenomena that may result in repeated changes in preferences, be it "passion", changes in the state of the world, or new information about the state of the world. To be sure, a lot of energy has been spent to determine the sensitivity of voting procedures to changes in voters' preferences, but those changes are assumed punctual and exogenous (see e.g. Nurmi, 2002, chapter 6).

The aim of this note is precisely to investigate how the volatility of voters' preferences affects the mutability of chosen policies. To do so, we focus on a committee that must decide on a one-dimensional issue by majority voting and whose members' preferences are volatile. In line with Tocqueville's presumption, we find that the policy chosen by majority voting is volatile and the more so the more volatile the preferences of the members of the committee. However, the volatility of the chosen policy is smaller than the volatility of any voter's ideal point, and a decreasing function of the size of the committee.

2. The whimsical committee

We assume that the committee consists of n ex ante identical individuals. It must decide on the value of a continuous variable x, by majority voting. Each committee member i has preferences that are single-peaked. In other words, we assume that each individual has an ideal value of x, labeled x_i^*. The main feature of our model is that each individual's ideal value of x is subject to random variations. We therefore assume:

$$x_i^* = X + u_{it} + v_t \qquad (1)$$

Where $X \in \Re$ is a constant, u_{it} and v_t are two normally-distributed independent random shocks with zero means and well-defined variances σ_u^2 and σ_v^2. v_t is a common shock affecting simultaneously the ideal points of all voters, whereas u_{it} only affects individual i. The former can therefore be interpreted as a fad, or as the result of a common shock that alters the situation of all committee members at the same time. The latter may be interpreted as a pure whim, i.e. as an idiosyncratic swing in voter i's preferences. It may also result from a more fundamental process, whereby committee members form their ideal outcomes on the basis of the evolution of their situation, which is itself subject to disturbances.[1] We assume for simplicity, that all voters' whims are drawn from the same distribution of probabilities.

As regards the timing of the decision process, we assume that all the uncertainty surrounding the realization of v_t and u_{it} is resolved before the vote takes place. This implies that the vote takes place in a deterministic setting. This also implies that, although voters are ex ante identical, they have different ideal values for x when they vote.

In those circumstances, the median voter theorem applies. Therefore, the outcome of the vote is the value of x that is preferred by the median voter, which we note x_M^*.[2] A complication arises here with respect to the usual application of the median voter theorem, as the identity of the median voter changes over time, due to the fact that voters' optimal outcomes vary randomly. If the number of committee members was infinite, this would not affect the result, as the median would equal the mean. However, as long as n is not infinite, the median depends on the realization of individual shocks. We must therefore investigate in details the stochastic properties of x_M^*.

First, the expected value of x_M^* is equal to the mean of x_i^*, which is equal to $X + v_t$. As the expected value of v_t is zero, we can conclude that the expected value of x_M^* is simply X.

[1] An illustration is an MP whose constituency is hit by an idiosyncratic shock, and who therefore adjusts his/her preferred tax rate. The same analogy would apply to a member of a monetary policy committee whose region's business cycle is specific. By contrast v_t would account for a symmetric shock affecting all constituencies or regions at the same time.

[2] A similar outcome would result in a representative democracy, since candidates running in the election would propose a platform that satisfies the median voter, as shown by Downs (1957).

But the outcome of the decision process cannot be summarized by its first moment. As the median voter's optimal outcome is stochastic, we must also, and chiefly, pay attention to the variance of the outcome, to which we may refer as a measure of the committee's whims. To do so, we must bear in mind that the median voter's favorite outcome is subject to two shocks. The first one, v_t, affects the mean of the distribution of voters. The second one, u_{it}, affects the distribution of voters around that mean. The variance of the chosen value of x is therefore the sum of the variance of v_t and of the variance of the median of a sample drawn from the distribution of u_{it}.[3] It therefore reads:

$$var\left(x_M^*\right) = \sigma_v^2 + \frac{\pi}{2n}\sigma_u^2 \tag{2}$$

As there are by definition at least two members in a committee, expression (2) clearly shows that the variance of the committee's decision is strictly smaller than the variance of its members' optimal outcomes. This takes us to our first proposition:

Proposition 1: A committee's whims are smaller than its members' whims.

Proof: The variance of any member's ideal outcome is given by $var(x_i^*) = \sigma_u^2 + \sigma_v^2$. Simple comparison of that expression with expression (2) reveals that the variance of x_M^* is strictly smaller than the variance of x_i^*, for any n greater than one.

A way to grasp intuitively this result is to recall that, since the committee makes its decisions by majority voting, the outcome of the decision process will be the median of the favorite outcomes of its members. To be sure, the median can be quite different from the mean and therefore subject to swings. However, the median voter can by definition never be the extreme voter. Majority voting therefore gives extremists no influence on the outcome of the vote, which accounts for the fact that the whims of the committee are smaller than the whims of its members.[4]

Nevertheless, the committee also has whims, since the outcome of the vote is stochastic. Expression (2) stresses that those whims even increase with the whims of voters, be they their individual whims or fads. However, it is also a decreasing function of the size of the committee. This takes us to our second proposition:

[3] The interested reader can refer to Kenney and Keeping (1962). It must be said that expression (2) rests on the expression of the variance of a large sample's median. Although no such expression exists for the variance of the median of a small sample, the estimates provided by Maritz and Jarrett (1978) show that our results can be extended to small samples. They therefore hold for small and large committees.

[4] To be more concrete, one may compare the outcome of majority outcome with the policy implemented by a dictator chosen among the members of the committee. The dictator may choose any value of x, even an extreme one. On the contrary, the extreme voter can never be the median voter, hence proposition 1.

Proposition 2: A committee's whims are larger the larger its members' whims and smaller the smaller its size.

Proof: From (2), $var(x_i^*)$ is clearly an increasing function of σ_u^2 and σ_v^2, but a decreasing function of n.

The intuition behind the first part of proposition 2 is straightforward. The whims of the committee originate from the whims of voters. If voters were not whimsical at all, i.e. if the variances of both u_t and v_t were zero, then the committee would not be whimsical either. In other words the committee is only whimsical because its members are subject to fads and whims. On the other hand, when the whims of voters increase, the committee becomes increasingly whimsical.

The second part of proposition 2 implies that the whims of committee members are mitigated by the size of the committee. The rationale for that result rests on the fact that, as the number of voters increases, they tend to be more evenly distributed around the mean. The medium voter is therefore likely to be closer to the mean, which accounts for the second part of proposition 2.

This intuition does not apply however to fads. One clearly sees that the variance of the outcome of the vote is greater or equal to the variance of v_t. In a committee whose sole source of whims would be fads, namely if $\sigma_u^2 = 0$, majority voting would have no stabilizing effect. In other words, $var(x_M^*)$ would be equal to $var(x_i^*)$.

Finally, one may also propose an alternative interpretation of our propositions that is reminiscent of Condorcet's jury theorem. Namely, if we assume that the members of the committee have to determine the true value of X on the basis of a noisy signal x_i^*, expression (2) implies that the committee will always be more accurate than any individual voter. It moreover implies that the committee will be more accurate the larger its size and, somewhat unsurprisingly, the more accurate its members. In the extreme case, if there were only voter-specific shocks and the size of the committee tended toward infinity, the committee would be perfectly accurate, which is consonant with Condorcet's theorem.

Concluding remarks

This note suggests that the outcome of majority voting in a committee is less variable, than the committee members' preferred outcomes, because majority voting mitigates their whims. In this sense, a committee can be said to be less whimsical than its members.

However, our reasoning rests on a very simple voting rule and a simple voting behavior. In particular, strategic voting is not allowed. It would therefore be interesting to consider that committee members can adopt more subtle voting behaviors and complex decision rules. Furthermore, the argument put forward here is purely positive. A natural extension would be to investigate the normative properties of majority voting when voters are whimsical and determine how it affects their welfare. This paves the way for future research.

References

Downs, A., 1957, An Economic Theory of Democracy, Harper and Row, New York.

Nurmi, H., 2002, Voting procedures under uncertainty. Springer Verlag, Berlin.

Schofield, N., 2002, “Representative democracy as social choice”. In K.J. Arrow, A.K. Sen and K. Suzumura (Eds.) Handbook of Social Choice and Welfare, volume 1, North-Holland, 425-455.

Tocqueville, A. de, 1835, Democracy in America.

Kenney, J.F., and E.S. Keeping, 1962, Mathematics of Statistics Part 2, Van Nostrand, Princeton.

Maritz, J.S., and R.G. Jarrett, 1978, A note on estimating the variance of the sample median, Journal of the American Statistical Association 73, 194-196.

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