







Pæper n° 71

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October 2004

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CAN BANKRUPTCY LAW DISCRIMINATE BETWEEN HETEROGENEOUS FIRMS WHEN INFORMATION IS INCOMPLETE? THE CASE OF LEGAL SANCTIONS

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JEL classification: G33, D82, D21.

Key words: bankruptcy, debt, bargaining, legal sanctions, law & economics, efficiency, incomplete information.

<u>Abstract</u>: We study conditions under which legal sanctions may lead to an efficient selection of heterogeneous investment projects. The bankruptcy code is a "primitive creditor system" and financial distress leads to an arbitration between private agreement and costly formal bankruptcy. We consider a standard debt contract between a bank and a small firm, both risk-neutral. There are two types of leveraged firms in the economy: profitable firms and non-profitable ones. Before the debt repayment time, firms arbitrate between continuation and voluntary liquidation (considered strategies may be pure or mixed). Non-profitable firms may be incited to pursue business because of limited liability. The legislator computes a collectively optimal level of legal sanctions that incites good firms to continue and bad ones to liquidate. For any level of legal sanctions, we show that costly formal bankruptcies may occur at equilibrium and the internalization of bankruptcy costs is impossible. Besides, when bankruptcy costs are not too high, an infinite level of legal sanctions may allow such a selection among heterogeneous firms. Nevertheless, because legal sanctions are bound to the level of the assets shortage, the legislator's action only leads to a second best optimum.

INTRODUCTION

When focusing on the bankruptcy process, one must concentrate on the time between the initial difficulties and the initiation of bankruptcy procedure. Indeed, in France for instance, 75% of firms which go bankrupt have no more assets at all, which leads to their immediate liquidation (Blazy and Combier, 1997). This fact means that numerous nonprofitable firms decide sub-optimal continuation, which finally reduces their chance of recovery. According to the same empirical study, the losses due to sub-optimal continuation explain the low level of creditors' repayment rates (less than 10%). Such sub-optimal continuation is found in four main situations:

- Entrepreneurs are risk lovers;
- Optimism induces biased anticipations;
- The bankruptcy procedure affects the firm's reputation;
- Priority rules and limited responsibility incite shareholders to continue any activity because there is always one state-of-the-world under which shareholders earn profit, even when the associated probability is low.

^(*) I am grateful to Jean-Philippe Spector for helpful comments and discussions. I also wish to thank Pierre-Marie Larnac, Jérôme Combier, Catherine Refait and participants at the international GDR symposium in money and finance at Lisbon, June 2000, and at seminars: Insee, Paris Dauphine University, and Lille University. All remaining errors are mine.

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Our approach insists more precisely on the last point because it characterizes any credit relationship, no matter the degree of risk aversion, the level of entrepreneurs' optimism or the existence of a possible effect on reputation.

Sub-optimal continuation must be understood here as the following behavior: a commercial or industrial activity is pursued by firms which are not the most profitable ones¹: from a collective point of view, the liquidation of their activity is necessary because it allows an efficient reallocation of resources towards more profitable firms. Creditors use more or less sophisticated discrimination mechanisms in order to make an optimal selection of their clients. Whatever their form (control mechanisms, Scott (1976), collaterals, Drukarczyk (1991) or incentive contracts, Jensen and Meckling (1976)), these mechanisms may be relatively costly or difficult to implement in practice, especially regarding medium and small firms. Without them, we can wonder if indirect environmental parameters discriminating between profitable and non-profitable firms exist. Particularly, the characteristics of bankruptcy codes may interfere in firms' decisions, even when they are solvent. From this point of view, when answering this question, we must distinguish between ex ante and ex post approaches of financial distress. The latter proposes different bankruptcy procedures applying to firms that are already in default. Aghion, Hart, and Moore (1992) and Aghion, La Porta Drago, Lopez-de-Silanes, and Moore (1997) propose an initial allocation of rights to the most senior creditors, with a resale option to junior claimants. This makes easier the vote procedure about the firm's future and maximizes the value of bankrupt firms. Bebchuk (1988) considers a similar serial auction procedure that facilitates [a] the firms' valuation and [b] the determination of the "residual claimant" (defined as the creditor who takes benefit from any increase in value of the firm). On the contrary, ex ante approaches deal with the effects of bankruptcy legislations on strategies taking place before a potential financial distress. Indeed, agents anticipate legal rules defined by the bankruptcy code: these affect firms' decisions, even if they are solvent (naturally, this approach is particular and must not occult the existence of other incentive mechanisms widely discussed by the theory of contracts). Here lawmakers can directly influence economic choices and, even, increase social efficiency. In particular, Berkovitch and Israel (1999) adopt this approach, by connecting economic fundamentals (market or bank-based systems) to bankruptcy laws' characteristics (does a creditor chapter coexist with a debtor chapter?). The authors show how bank-based economies, such as Germany, only need a creditor bankruptcy chapter (the bankruptcy procedure is initiated by creditors). Indeed, in such economies, relationships between firms and banks are so close that the debtor knows a lot about the information acquisition technologies of its creditor. Taking this into account, a debtor chapter might be used strategically by the debtor in a sub-optimal way. Here, an optimal bankruptcy law should include a creditor chapter only, in order to maximize the value of firms². On the contrary, in market-based economies, such as the United-States, an optimal bankruptcy law should include both chapters: a creditor chapter (cf. "chapter 7" of the 1978 U.S. "Bankruptcy Reform Act") and a debtor chapter (cf. "chapter 11").

¹ This does not always mean that these firms are non-profitable: they are simply not the best ones and a reallocation of economic resources could increase social surplus.

 $^{^{2}}$ Social efficiency implies the maximization of this value: a firm must continue if and only if its continuation value exceeds its liquidation value.

Our approach is similar to Berkovitch and Israel's model because it focuses on the *ex ante* effects of the Law and does not consider conflicts among creditors³. However, we deviate from their analysis by focusing on the arbitration between continuation and liquidation, which takes place before a possible financial distress. This is a key question because choosing sub-optimal continuation may induce an increase of losses. On the contrary, choosing sub-optimal liquidation may induce opportunity costs. Taking that point of view, the main difficulty is to enforce the optimal timing of bankruptcy, and to avoid sub-optimal delays: this is the aim of our contribution.

Many incentive legal tools are at the disposal of lawmakers in order to change the behavior of firms before financial distress occurs: indeed, bankruptcy codes include many parameters, such as the identity of the bankruptcy initiator, the level of legal sanctions, the nature of priority rules⁴, the deviations from the latter, the bargaining power of each party, the role of bankruptcy officials and of tribunals during the reorganization process, (etc.). When focusing on the optimal timing of bankruptcy, we shall restrict our analysis to the incentive power of legal sanctions on heterogeneous investment projects. In case of sub-optimal decisions, an efficient bankruptcy law may enforce legal sanctions. This approach allows a distinction between firms (which must be evaluated with regards to their economic profitability) and managers⁵ (who can be sanctioned with regard to their implication in financial difficulties). It is usual to separate criminal sanctions, which apply in case of fraudulent behavior, from financial sanctions, which apply in the case of management errors. It is usual to consider that delay in filing is closer to the second case: from now on, we shall name such financial sanctions "legal sanctions", which means the possibility of being held personally liable for the firm's debts⁶. It is advisable to note that the incentive power of legal sanctions towards more or less profitable investment projects is not trivial if information is incomplete and if financial distress can be treated in an informal way. We shall see notably that non-profitable firms having opted for the continuation can sometimes escape from legal sanctions: this is the case when the bank believes they may be profitable ones. Here, bad projects can "bluff" and, finally, avoid bankruptcy. Here, legal sanctions do not apply and cannot play their incentive role anymore. Taking that point into account, the aim of our article is to analyse the cases under which lawmakers achieve an efficient discrimination between projects and, if possible, to determine the associated optimal level of legal sanctions.

The paper is organized as follows. Section I presents the model. The next two sections follow a backward induction procedure: section II discusses the arbitration between private agreement and formal bankruptcy in case of financial distress and for any level of legal sanctions. Section III analyses the initial "continuation vs. liquidation" decision, which takes place before the realization of the investment project, and calculates the optimal level of legal sanctions under mixed strategies: it is then possible to formalize the legislator's objective function and to determine if his action leads to a first best Pareto-optimum.

³ Franks and Nyborg (1996) propose a model where two creditors bargain under the U.K. Insolvency Code.

⁴ White (1989) defines three basic priority rules: the "Me first rule", the "Last lender first rule" and the "Equal priority rule". Usual bankruptcy codes mix each of them.

⁵ This approach has prevailed in France since 1967.

⁶ Managers may also be barred from managing any other firm, but this fact is off the point here.

I. THE FRAMEWORK OF THE MODEL

We consider an economy made up of small firms. Entrepreneurs⁷ live for a period, which covers the discrete interval [t, t+1]. None of them owns an initial capital. Consequently, for each of them, the start of their new business means borrowing an amount $D_{t-\epsilon}$ at time $(t-\epsilon)$, covering the expenses necessary for the investment project. The contractual interest rate is noted $i_{t-\epsilon}$, which is less than the usurious interest rate $i_{t-\epsilon}^{M}$. Let's consider a debt contract⁸ between a bank (monopoly) and one of these small firms (which can be assimilated to its manager-shareholder⁹). Both agents are risk-neutral. The debt contract is standard: it specifies a transfer of control from the firm to the bank if financial distress occurs (Aghion and Bolton, 1992). This transfer takes place within a legal procedure (the bankruptcy code), which ends with the liquidation of the firm (which corresponds to the "primitive creditor system" defined by Franks and Nyborg, 1992): the proceeds of this liquidation, minus bankruptcy costs, are used to pay off the bank, before the firm, according to the Absolute Priority Rule. At last, it must be noted that the sole destination of leverage is the acquisition of capital (cumulated investment), noted at time t (I_t^c); therefore, both variables ($D_{t-\epsilon}$ and I_t^c) are identical.

There are two types of firms in the economy whose efficiency is indexed by $(k)^{10}$. The type "k = 0" (respectively "k = 1"), whose probability is p_0 (resp. $p_1 = 1 - p_0$), characterizes good (resp. bad) firms and is public information: p_0 can be interpreted as the proportion of good investment projects in the economy. The firm discovers its real type during a learning process¹¹, which takes place between times $(t - \varepsilon)$ and (t), whereas it remains unknown to the bank (a systematic audit procedure would not be a realistic hypothesis because of the small size of the firms considered here). Finally, our approach describes a classical situation where information is asymmetric and incomplete between the lender and the borrower.

The return per unit of capital of the project is designed by a random variable $(\tilde{x}|k)$, whose mathematical expectation depends on the value of parameter (k). The realization of this variable (x_{t+1}) equals (x), strictly positive, with probability p(x|k) (successful project) and zero otherwise. The level of the operating result realised at time t+1 can be inferred from the return per unit of capital of the firm, multiplied by the cumulated investment at

⁷ From now on, roughly speaking, we shall name them the "managers-shareholders".

⁸ The contracting process is not described here and the contractual interest rate is exogenous. A way of extending our model would be the following: at time $(t-\varepsilon)$, agents arbitrate between entrepreneurship (which implies leverage) and wage-earning (summarized by a reservation utility, corresponding to the present sum of expected wages until the end of times). Under this presentation, the bank would compute the optimal interest rate, by maximizing its expected profit under the following participation constraint: expected profits coming from entrepreneurship must be greater than the reservation utility.

⁹ Hence, in the small firm, there is no conflict of interests between managers and shareholders who are supposed to be the same person.

¹⁰ We consider a population made up of two groups of "clones": each firm can be considered as the representative individual of its own group. Nature decides the firms' group with probabilities p_0 and $1-p_0$.

¹¹ This learning process is not described here. Like Blazy (1999), we could imagine a Bayesian revision process based on the realization of a particular signal (such as results recently observed, for instance). Nevertheless, taking this into account would not change the results of our model.

time t+1 (I_{t+1}^c). As a consequence, the operating result of the firm is also a random variable $(\tilde{R}_{t+1}|k)$, whose realization (r_{t+1}) equals, at the end of the period, ($x \cdot I_{t+1}^c$) with probability p(x|k) and zero with probability 1 - p(x|k). Besides, between times t and t+1, capital obsolescence (at rate ω) is given by:

$$I_{t+1}^{c} = \frac{I_{t}^{c}}{1+\omega} \quad \text{with} \quad \omega > 0 \tag{1}$$

Contrary to the bad state of the world, we suppose that the good state (which implies the success of the project) preserves the firm from any risk of financial distress. Therefore, under this hypothesis, and remembering that the manager – whose life ends at time t+1 – always liquidates his firm at the end of the period¹², the operating result realized in case of success $(x \cdot I_{t+1}^c)$ plus the proceeds of the liquidation of the firm at time t+1 (V_{t+1}^L) always cover the overall debt charge (i.e. the amount of the loan plus interest payment, whose maximum level is reached when $i_{t-\epsilon}$ equals the usurious interest rate). This can be described as follows:

$$\mathbf{x} \cdot \mathbf{I}_{t+1}^{c} + \mathbf{V}_{t+1}^{L} \ge \left(\mathbf{l} + \mathbf{i}_{t-\varepsilon}^{M}\right) \cdot \mathbf{D}_{t-\varepsilon}$$

$$\tag{2}$$

"Type k = 0" firms are the most efficient ones (p(x|0) > p(x|1)) and, hence, guarantee the best exploitation of economic resources. So, from a collective point of view, it would be efficient to help these firms to continue their activity and incite the others to liquidate. Following this idea, we shall suppose that probabilities of success, p(x|0) and p(x|1), have not the same value: without leverage, only good firms should continue whereas bad ones should choose voluntary liquidation¹³. Firms' liquidation value is supposed to be public information: the bank knows the parameter ρ , which characterizes the drop in value due to assets resale. At time t, the liquidation value of the firm can be formalized as follows:

$$V_t^L = \frac{I_t^c}{1+\rho} \quad \text{with} \quad \rho > 0 \tag{3}$$

Without leverage, remembering that continuation represents the optimal choice for "type k = 0" firms, the probability of success should respect the following condition (where E(.) is the expectation operator and supposing a zero discount rate):

Continuation value $(k = 0) \ge$ liquidation value (k = 0)

$$\Leftrightarrow E(\widetilde{R}_{t+1}|0) + V_{t+1}^{L} \ge V_{t}^{L} \iff E(\widetilde{X}|0) \cdot \frac{I_{t}^{c}}{1+\omega} + \frac{I_{t}^{c}}{(1+\omega) \cdot (1+\rho)} \ge \frac{I_{t}^{c}}{1+\rho}$$
$$\Leftrightarrow E(\widetilde{X}|0) \ge \frac{\omega}{1+\rho} \iff p(x|0) \ge \frac{\omega}{x \cdot (1+\rho)}$$

¹² This implies that there is no altruism between the successive generations of managers.

¹³ The adjective « voluntary» designates a liquidation that has been decided by the manager: it does not occur under bankruptcy.

On the contrary, the expected profitability of a "type k = 1" firm should provide incentives for its voluntary liquidation. Finally, we obtain the following condition:

$$p(x|0) \ge \frac{\omega}{x \cdot (1+\rho)} > p(x|1)$$
(4)

Consider now a leveraged firm. At time t, its manager chooses between continuation until t+1 and voluntary and immediate liquidation. Of course, the fact that a firm chooses continuation represents a signal for the bank: at time t+1, considering the decision taken at time t, the bank calculates a revised probability about the firm's type: $p(0|C_t)$ for a "k = 0" firm and $1 - p(0|C_t)$ for a "k = 1" firm (C_t refers to the decision of continuation taken at time t). In case of failure (x_{t+1} = 0), this probability is revised for a second time and $p(0|C_t)$ becomes $p(0|C_t; x_{t+1} = 0)$. This double probability revision is made by the bank during the debt renegotiation process, which takes place at time t+1 (see section II for a description).

From a collective point of view, only "type k = 0" firms should use economic resources (the others, less profitable, should then liquidate capital they do not use in an optimal way)¹⁴. Nevertheless, because the investment project only yields returns at time t+1, liquidation decided at time t necessarily implies financial distress ($V_t^L = D_{t-\varepsilon}/(1+\rho)$) is always lower than $D_{t-\varepsilon}$). Hence, because of limited responsibility, "type k = 1" managers (whose personal capital contribution is supposed to be equal to zero) may, all the same, choose continuation in order to avoid financial distress at time t. Here, legal sanctions can reduce this sub-optimal behavior.

In most developed countries, pecuniary sanctions apply to mismanagement cases, including sub-optimal continuation (i.e. civil and penal sanctions are excluded from our model¹⁵): funds are transferred from the manager's personal wealth towards the firm's funds, in order to repay creditors. These sanctions (S_{t+1}) apply to distressed firms which decided to continue at time (t), whereas they are in fact unprofitable, if bankruptcy is triggered at time (t+1) (bankruptcy costs help to discover the firm's real type). The basis of legal sanctions is the manager's personal possessions (P_{t+1}). Here, two important remarks have to be made. Firstly, these possessions are specific (a house, for instance) and, then, cannot be used to finance business: leverage is the sole financial resource here. Secondly, the manager's personal wealth appears between times (t) and (t+1), so that it cannot be included in the debt contract, as collateral, at time ($t - \varepsilon$)¹⁶.

The maximum level of legal sanctions (S_{t+1}^{M}) is limited by the lowest value between the manager's personal possessions (P_{t+1}) (which are supposed to be infinite, for simplification) and the spread between the total amount to be repaid (i.e. debt repayment, $(1+i_{t-\epsilon}) \cdot D_{t-\epsilon}$,

¹⁴ For instance, we could imagine the resale of capital stock from "type k = 1" firms to "type k = 0" ones, with a debt transfert to these new businesses.

¹⁵ Civil and penal sanctions only apply to the most severe cases such as the pursuit of self-interests or deceitful acts, which is out of the scope of our analysis.

¹⁶ Nevertheless, we shall suppose that all agents perfectly anticipate at time (t) the level of the manager's personal wealth.

plus bankruptcy costs, C_{t+1}^{F} ¹⁷) and the firm's available wealth. The latter covers the operating result (which equals zero in case of bankruptcy) plus the liquidation value of the firm at time t+1 (V_{t+1}^{L}) (because the legal procedure corresponds to a "primitive creditor system", i.e. the only legal solution is the liquidation of the firm). S_{t+1}^{M} respects the following relation (supposing that the legal procedure generates bankruptcy costs, which proportionally increase with the size of the firm (at rate β_{F}), indirectly reflected by the level of I_{t+1}^{c}):

$$S_{t+1}^{M} = min \Big(P_{t+1}; (1 + i_{t-\epsilon}) \cdot D_{t-\epsilon} + C_{t+1}^{F} - V_{t+1}^{L} \Big)^{18}$$
(5)
with $C_{t+1}^{F} = \beta_{F} \cdot I_{t+1}^{c}, \ \beta_{F} \in [0;1]$

Legal sanctions are enforced during the bankruptcy procedure. Notice that the latter is not automatically triggered off. It only applies in case of failure of the private and informal debt renegotiation, initiated by the debtor (i.e. the firm).

Hence, the model sequence can be described as follows (figure 1). At time t+1, the operating result of the firm may be equal to zero with probability 1 - p(x|k) and, then, be insufficient to cover debt charges. Taking this point of view, we adopt an exogenous definition of financial distress (as Kim, Ramaswamy and Sundaresan, 1993), which can be associated to a strict liquidity constraint. This approach characterizes, for instance, the French legal definition of financial distress, which is "declared" when the firm can no longer repay its short-term engagements with its liquid assets (see the third article of the 1985 Bankruptcy Code). In order to avoid bankruptcy when financial distress occurs, the debtor makes a paying off proposal to the bank, which amounts to A_{t+1} and has to be paid when the firm is liquidated (the firm is *always* liquidated at time t+1, because of end of times). If the bank accepts the firm's proposal, and receives A_{t+1} , the manager earns the liquidation value of its firm minus the paying off: $V_{t+1}^{L} - A_{t+1}$. In case of refusal, a legal procedure (formal bankruptcy, strictly speaking) is triggered off and the firm is liquidated¹⁹. As usual, bankruptcy costs are paid before the bank's claim²⁰. The main characteristic of these costs (among which expert and evaluation fees) is that they produce information: they reveal the firm's type $(k = 0 \text{ or } k = 1)^{21}$. Webb (1987) describes them as "verification costs". If it appears that the firm's type is k = 1, the continuation decision taken at time t was collectively

¹⁷ For each firm, bankruptcy costs are supposed to be lower than its liquidation value. Indeed, the literature dealing with the empirical measures of bankruptcy costs shows that they are relatively low, when compared to the liquidation value of firms. Highest measures are recorded in White (1989) and Baxter (1967). Baxter estimates the share of direct personal bankruptcy costs between 19.9% and 25.7% of the liquidation value. According to the author, these rates must decrease when bankruptcy procedures apply to firms.

¹⁸ This relation means that bankruptcy costs must be considered as new claims that increase the firm's overall liabilities.

¹⁹ There is no vote from the creditors. The game described here is closer to the French legal procedure, under which the liquidation is decided by the Court.

²⁰ In case of bonus, the financial surplus would belong to the manager-shareholder. Nevertheless, this case does not hold here: the amounts recovered under bankruptcy (including legal sanctions, if necessary) are supposed to be lower than the bank's claim.

²¹ The model could be complexified, by introducing hazard into the firms' type discovery process, when formal bankruptcy is triggered off.

sub-optimal and justifies the application of legal sanctions, the equilibrium level and appropriateness of which remain to be determined.





II. PRIVATE AGREEMENT VS. FORMAL BANKRUPTCY

We use a backward induction procedure to solve this game. At time t+1, if financial distress occurs (i.e. the operating result is zero) and if the informal debt renegotiation does not succeed²², a bankruptcy procedure is triggered off. The bank is never repaid in full if legal sanctions are not imposed. Indeed, under our hypothesis, the following inequality always prevails: $(1 + i_{t-\epsilon}) \cdot D_{t-\epsilon} > V_{t+1}^L - C_{t+1}^F$.

When financial distress occurs, the firm makes a paying off proposal (A_{t+1}) to the bank in order to avoid bankruptcy. Of course, "type k = 0" managers know that their firms will earn nothing in case of bankruptcy. Nevertheless, legal sanctions cannot be imposed on them (the failure of their business is due to bad luck, not to the voluntary continuation of a nonprofitable activity). Then, "type k = 0" firms prefer a private debt renegotiation to a formal bankruptcy if the amounts recovered within the private agreement are positive or equal to zero²³. These amounts equal the proceeds of the voluntary liquidation of the firm (V_{t+1}^L) minus the contractual amount offered to the bank (A_{t+1}) :

"Type k=0" firms prefer private debt renegotiation to formal bankruptcy if and only if:

$$A_{t+1} \le V_{t+1}^L \tag{6}$$

Taking their own points of view into account, "type k = 1" managers know they are exposed to personal sanctions (S_{t+1}) if the bank refuses their offer. Hence, they are disposed to propose a greater amount to the bank, so that their decision rule becomes the following:

"Type k = 1" firms prefer private debt renegotiation to formal bankruptcy if and only if:

$$A_{t+1} \le V_{t+1}^{L} + S_{t+1}$$
(7)

In the same way, the bank only accepts offers that exceed the amounts that are expected to be recovered under bankruptcy. In that case, on one hand, bankruptcy costs reduce the value of the firm, but, on the other hand, financial sanctions can increase the level of repayment if the firm is of type "k = 1". However, unlike the firm, the bank does not know the real value of k. Therefore, one could suppose that the bank's behavior is based on the revised probability $p(0|C_{t,x_{t+1}=0})$. This would be incomplete. Indeed, the firm's proposal (A_{t+1}) is another signal that enables the bank to revise its beliefs about the type of the firm for a second time. Hence, after proposal has been made, we obtain a newly revised probability, noted $p(0|C_{t,x_{t+1}=0;A_{t+1})$, whose determination will be described later.

²² We consider a "one shot" renegotiation process: the bank cannot make a counterproposal. This assumption is quite realistic within the frame of the French legislation, under which managers are obliged to file for bankruptcy within 15 days of the date at which the firm becomes unable to pay debts when they are due.

²³ The underlying assumption is that agents always choose informal renegotiation when both solutions yield identical amounts.

The decision rule of the bank can be written as follows :

The bank accepts private debt renegotiation if and only if:

$$A_{t+1} \ge V_{t+1}^{L} - C_{t+1}^{F} + (1 - p(0|C_{t}; x_{t+1} = 0; A_{t+1})) \cdot S_{t+1}$$
(8)

The second term of this inequality represents the threshold under which the bank refuses the renegotiation and triggers bankruptcy. We shall name it the "bank's acceptance threshold" in the rest of the paper. In particular, it depends on the amounts repaid if the firm is of type "k = 1", including legal sanctions. Therefore, the bank's acceptance threshold is a decreasing function of the newly revised probability $p(0|C_t;x_{t+1}=0;A_{t+1})$.

It is necessary to distinguish between two cases in order to describe the debt renegotiation mechanism, the induced probability revision process, and the bankruptcy trigger decision. The first one characterises a "severe legal system" (where legal sanctions are relatively high: $S_{t+1} \ge C_{t+1}^{F}$), whereas the second one describes a "moderate legal system" (where legal sanctions are somewhat less drastic: $S_{t+1} < C_{t+1}^{F}$).

II.1. Bargaining under a "severe legal system": $S_{t+1} \ge C_{t+1}^F$

The firm derives benefit from its informational advantage and from the possibility of initiating the debt renegotiation process. This bargaining between the firm and the bank leads to an internalisation of bankruptcy costs.

PROPOSITION 1. It is possible to define two (separating or pooling) equilibria, which depend on the initial value of $p(0|C_t;x_{t+1}=0)$ (this probability has to be revised for a second time, after the communication of A_{t+1} : then, $p(0|C_t)$ becomes $p(0|C_t;A_{t+1})$).

Proof. Looking at figure 2, it appears that there is a threshold value for the probability of being a "type k = 0" firm: this (noted \hat{p}_0) is characterized by the equality between the bank's acceptance threshold and the liquidation value of the firm (cf. point **2**, figure 2).

$$V_{t+1}^{L} - C_{t+1}^{F} + (1 - \hat{p}_{0}) \cdot S_{t+1} = V_{t+1}^{L}, \text{ that is } \hat{p}_{0} = 1 - \frac{C_{t+1}^{F}}{S_{t+1}}$$
(9)

This threshold value plays a central part in the probability revision process:

Figure 2: Acceptance areas for the private agreement when $S_{t+1} \ge C_{t+1}^F$



<u>Comment</u>: The main axis (in bold) indicates all possible values for the amount A_{t+1} , which is proposed by the firm to its bank, in order to avoid formal bankruptcy. Areas covered by the three small arrows specify acceptable values of A_{t+1} from the (type "k=0" or "k=1") firm's or bank's point of view. Notice that the bank's acceptance threshold depends on the value of the new revised probability of being a "type k=0" firm. The direction of arrows designates an increasing gain derived from the private agreement (these arrows are, of course, in the opposite direction depending on whether the agent considered is the firm or the bank). Hence, it is only possible to reach a contractual agreement within the overlapping areas.

- If $p(0|C_t;x_{t+1}=0) \in [0; \hat{p}_0[$ (i.e. *low proportion of profitable firms* at time t+1²⁴) : the bank's acceptance threshold (before the new revision of its beliefs) is located at the right side of V_{t+1}^{L} (beyond the point **2**). In that case, offers from "type k = 0" firms are never accepted by the bank. Indeed, "type k = 0" firms and the bank's acceptance areas do not overlap. Hence, acceptable offers only come from "type k = 1" firms: taking this into account, the bank revises its beliefs $p(0|C_t;x_{t+1}=0)$ so that $p(0|C_t;x_{t+1}=0;A_{t+1})=0$. The new bank's acceptance threshold (defined after this second revision) is now located at point **3**: the firm proposes an amount A_{t+1}^* , equal to this threshold, which is always accepted by the bank.

The renegotiation equilibrium separates the two populations of firms: "type k = 0" firms prefer formal bankruptcy to private agreement, whereas "type k = 1" firms avoid bankruptcy by paying the following amount:

$$A_{t+1}^* = V_{t+1}^L - C_{t+1}^F + S_{t+1} \quad (\text{if } k = 1)$$
(10)

It must be noticed that "type k = 1" cannot "bluff", by proposing a lower amount than V_{t+1}^{L} (at the left side of point **2**), in order to hide their true type and, then, to pay less. Indeed, such a proposal would not be bearable. The bank could not distinguish between the different types of firms any longer and would not be able to revise its beliefs another time: $p(0|C_t:x_{t+1}=0;A_{t+1})$ would remain equal to $p(0|C_t:x_{t+1}=0)$ (strictly lower than \hat{p}_0 , under the considered case). The bank's acceptance threshold would remain greater than V_{t+1}^{L} (at the right side of point **2**) and the firm's offer would be rejected.

²⁴ From the bank's point of view.

- If $p(0|C_{t}:x_{t+1}=0) \in [\hat{p}_0;1]$ (i.e. *high proportion of profitable firms* at time t+1): the probability of being a "type k = 0" firm is relatively high and the bank's acceptance threshold (before the revision of its beliefs) is located somewhere between points **0** and **2** (figure 2). A "type k = 1" firm can take advantage of this situation, proposing an amount compatible with the "type k = 0" offers. Here, the renegotiation equilibrium cannot separate good from bad firms. Indeed, both types can reach the bank's acceptance threshold: each of them proposes an amount equal to this threshold and this offer is always accepted by the bank (whose beliefs cannot be revised another time: $p(0|C_{t}:x_{t+1}=0:A_{t+1}) = p(0|C_{t}:x_{t+1}=0)$). The contractual amount equals:

$$A_{t+1}^{*} = V_{t+1}^{L} - C_{t+1}^{F} + \left(1 - p(0|C_{t}; x_{t+1} = 0)\right) \cdot S_{t+1} \quad (\forall k)$$
(11)

Hence, depending on the initial value of $p(0|C_t, x_{t+1}=0)$, the renegociation debt process leads to two different – pooling or separating – bargaining equilibria. This result only prevails when the legal system is "severe".

II.2. Bargaining under a "moderate legal system": $S_{t+1} < C_{t+1}^F$

PROPOSITION 2. Under a "moderate legal system", the renegociation debt process always leads to a pooling equilibrium.

Proof. When legal sanctions are lower than bankruptcy costs, the bank's acceptance threshold is always below the liquidation value of the firm, whatever the level of $p(0|C_t)$ (figure 3).





Like the "severe" legal system with a "high" proportion of profitable firms, it is not possible to separate bad from good firms. As a matter of fact, the bank's acceptance threshold is located somewhere between points **0** and **2** (figure 3). Then, the firm, whatever its type, proposes an amount A_{t+1}^* equal to this threshold, which is always compatible with its own acceptance area. Formal bankruptcy is never triggered off (bankruptcy costs are internalized) and A_{t+1}^* equals the following expression:

$$A_{t+1}^{*} = V_{t+1}^{L} - C_{t+1}^{F} + \left(1 - p(0|C_{t}; x_{t+1} = 0)\right) \cdot S_{t+1} \quad (\forall k)$$
(12)

Notice that the second revision of bank's beliefs is impossible because the signal A_{t+1}^* may come from any type of firm and, then, brings no new information.

II.3. Financial distress resolution: a synthesis

At time t+1, the amounts recovered by each party and the type of financial distress resolution depend on the value of probability $p(0|C_t, x_{t+1}=0)$, especially in comparison with \hat{p}_0 (this one increases with the level of sanctions). Table 1 summarizes these different cases:

	$S_{t+1} \ge C_{t+1}^F$		s c ^F	
	$p(0 C_t;x_{t+1}=0) < \hat{p}_0$	$p(0 C_t;x_{t+1}=0) \ge \hat{p}_0$	$S_{t+1} < C_{t+1}$	
Firm (k=0)	0	C_{t+1}^{F} (**)	C ^F _{t+1}	
Firm $(k = 1)$	$C_{t+1}^F - S_{t+1}^{(*)}$	$-(1-p(0 C_t;x_{t+1}=0)) \cdot S_{t+1}$	$-(1-p(0 C_t;x_{t+1}=0)) \cdot S_{t+1}$	
Bank	$\begin{split} & V_{t+1}^L - C_{t+1}^F \ (k=0) \\ & V_{t+1}^L - C_{t+1}^F + S_{t+1} \ (k=1) \end{split}$	$ \begin{array}{c} V_{t+1}^{L} - C_{t+1}^{F} \\ + (1 - p(0 C_{t}; x_{t+1} = 0)) \cdot S_{t+1} \end{array} (\forall k) \end{array} $	$ \begin{array}{c} V_{t+1}^{L} - C_{t+1}^{F} \\ + (1 - p(0 C_{t}; x_{t+1} = 0)) \cdot S_{t+1} \end{array} (\forall k) \end{array} $	
Financial distress resolution	Formal bankruptcy (k=0) Private agreement (k=1)	Private agreement (∀k)	Private agreement (∀k)	

<u>Table 1</u>: Amounts recovered and financial distress resolution (formal bankruptcy *vs.* private agreement)

(*) This amount is negative. However, the loss is smaller outside bankruptcy because bankruptcy costs are internalised during the private renegotiation process.

(**) This amount is positive or equal to zero if $p(0|C_t,x_{t+1}=0) \ge \hat{p}_0$.

"Type k = 0" firms anticipate that they will not pay legal sanctions in case of bankruptcy. Therefore, they are less inclined to initiate private agreements than "type k = 1" firms: their repayment offers are smaller. Because of the presence of non-profitable firms in the economy, these offers may be insufficient to avoid bankruptcy. This happens when $p(0|C_t, x_{t+1}=0) < \hat{p}_0$. On the contrary, when the proportion of profitable firms is high $(p(0|C_t, x_{t+1}=0) \ge \hat{p}_0)$, a "type k = 1" firm may take advantage of the bank's information deficit and propose a smaller repayment offer $(V_{t+1}^{L} - C_{t+1}^{F} + (1 - p(0|C_{t}, x_{t+1} = 0)) \cdot S_{t+1})$ than the one that would prevail in the case of perfect and complete information $(V_{t+1}^{L} - C_{t+1}^{F} + S_{t+1})$. In that case, the bank cannot distinguish between different types of firms. Nevertheless, this story only holds if "type k = 1" firms actually choose continuation at time t. On that point, the legislator could conduct a discriminating and collectively efficient action, by inciting "type k = 1" firms to liquidate and "type k = 0" firms to continue at time t. If the legislator could reach this objective (which would then become public information), probability $p(0|C_t, x_{t+1}=0)$ would be equal to one and the repayment offer could be written as follows: $V_{t+1}^{L} - C_{t+1}^{F}$. Examining the "continuation vs. liquidation" decision and the resulting change in the bank's beliefs is necessary in order to understand the nature of the legislator's interference.

III. PROBABILITY OF CONTINUATION: THE INCENTIVE POWER OF LEGAL SANCTIONS

At time t, the decision of voluntary liquidation implies financial distress: liquid assets $(V_t^{L} = D_{t-\epsilon}/(1+\rho))$ do not cover the amount to be repaid (outstanding debts: $D_{t-\epsilon})^{25}$. As described before, the firm makes an offer (A_t) to the bank in order to avoid bankruptcy.

PROPOSITION 3. The renegotiation process is simpler than the one that takes place at time (t+1) and always leads to the internalization of bankruptcy costs.

Proof. At time t, no firm may incur legal sanctions, because sub-optimal continuation²⁶ is not defined at this time ("continuation" implies waiting until time t+1). Therefore, a firm, whatever its type, prefers private debt renegotiation to formal bankruptcy if and only if:

$$A_t \le V_t^L \tag{13}$$

As for the bank, it accepts this offer if and only if the following condition holds (remembering that bankruptcy costs defined at time t are equal to $C_t^F = \beta_F \cdot I_t^c$, with $I_t^c = D_{t-\epsilon}$):

$$A_t \ge V_t^L - C_t^F \tag{14}$$

Hence, the firm, which initiates the renegociation process²⁷, offers an amount equal to $V_t^L - C_t^F$ and recovers the internalised bankruptcy costs C_t^F : this quantity is the amount recovered in case of liquidation at time t.

Here again, the analysis of the continuation decision rule must distinguish between "severe" and "moderate" legal systems as defined above.

²⁵ If there were no drop in the value of assets during liquidation (ρ =0), the firm would not be in financial distress anymore, which would have a significant influence on equilibria. Indeed, at time t, the renegotiation process, which leads to condition (15), would disappear. The new condition for continuation would simply become: "the firm continues if and only if its expected profit is greater or equal to zero". We may notice that this condition is almost always verified (see expected profits at time t). The sole exception concerns "type k=1" firms, under the "severe" legal system with a "low" proportion of profitable firms at time t+1 (see above). In that case, the expected profit of non-profitable firms might be negative when the investment project fails. It is thus possible to prevent bad firms from choosing continuation by modifying the level of legal sanctions.

prevent bad firms from choosing continuation by modifying the level of legal sanctions. ²⁶ We use the terminology of "sub-optimal continuation" because non-profitable firms decide to continue, whereas economic resources could be exploited more optimally by profitable ones.

²⁷ Naturally, the fact that the firm plays first increases its bargaining power. This advantage is justified because the firm observes financial distress before the bank and, therefore, is able to assess the opportunity for informal renegociation. Under the "one shot bargaining" assumption (see above), a complementary approach would consist in considering a bargaining process initiated by the bank. Nevertheless, this would not be quite realistic: for instance, the 1984 French law, dealing with out of court arrangements, provides for bargaining processes initiated only by managers (sometimes after having been warned by the tribunal). Yet, it must be noticed that initiating the out-of-court arrangement is slightly different from initiating the bargaining process itself.

III.1. Severe legal system: $S_{t+1} \ge C_{t+1}^F$

In the severe legal system, the two cases, $p(0|C_t;x_{t+1}=0) < \hat{p}_0$ and $p(0|C_t;x_{t+1}=0) \ge \hat{p}_0$, must be analyzed separately, because each of them implies a different renegotiation equilibrium. Especially, only the first case leads to a separating equilibrium, from the bank's point of view.

III.1.1. Low proportion of profitable firms: $p(0|C_t, x_{t+1}=0) < \hat{p}_0$

As we shall see, this case is the simplest one. Besides, it does not hold anymore when the legislator defines an efficient level of legal sanctions.

Firms assign probabilities to each possible action: by acting like that, they adopt mixed strategies²⁸ and compute the optimal level of probability of continuation $p(C_t|k)^*$. Each "type (*k*)" firm calculates its expected profit, using probabilities $p(C_t|k)$ and $1 - p(C_t|k)$ to weigh the gains associated with each possible action (continuation and liquidation). Then, conditional probabilities $p(C_t|0)$ and $p(C_t|1)$ are respectively the maximization variables of the "type k = 0" and "type k = 1" firms' programs.

Separating equilibrium means that when financial distress occurs, the amounts recovered by the firm depend on its type (see above: table 1): consequently, the formalization of each firm's program strongly depends on its type.

A. <u>The "type k = 1" firms' programs</u>

At time t, "type k = 1" firms decide continuation until time t+1 with probability $p(C_t|1)$. In that case, if the investment project succeeds (with probability p(x|1)), the firms' earnings amount to the operating result $(x \cdot I_{t+1}^c)$, plus the proceeds of liquidation occurring at the end of times (V_{t+1}^L) , minus financial charges, $(1 + i_{t-\epsilon}) \cdot D_{t-\epsilon}$. In case of failure, firms earn $C_{t+1}^F - S_{t+1}$ (see above, table 1). When liquidation is decided at time t, with probability $1 - p(C_t|1)$, we know that firms recover bankruptcy costs C_t^F , which are internalized during the renegociation process. Knowing this, each firm calculates $p(C_t|1)^*$ by resolving the following program (where Π symbolizes the expected profit which is weighted by the probabilities of continuation and liquidation):

$$\max_{(p(C_t|l))} E_t(\Pi|l) = p(C_t|l) \cdot \left(p(x|l) \cdot \left(x \cdot I_{t+1}^c + V_{t+1}^L - (1 + i_{t-\varepsilon}) \cdot D_{t-\varepsilon} \right) + (1 - p(C_t|l)) \cdot C_t^F \right) + (1 - p(C_t|l)) \cdot C_t^F$$
(15)

PROPOSITION 4. The legislator can define a level of legal sanctions that dissuades nonprofitable firms from continuing their activity.

²⁸ These probabilities can be considered as the beliefs of players (here, the bank) about opponents' (the firm) possible actions (Bierman and Fernandez, 1998).

Proof. The resolution of program (15) leads to:

$$\frac{dE_{t}(\Pi|1)}{dp(C_{t}|1)} = p(x|1) \cdot \left(x \cdot I_{t+1}^{c} + V_{t+1}^{L} - (1 + i_{t-\varepsilon}) \cdot D_{t-\varepsilon}\right) + (1 - p(x|1)) \cdot \left(C_{t+1}^{F} - S_{t+1}\right) - C_{t}^{F} \quad (16)$$

This expression may be positive or negative. Here, the legislator has a power of incentive on the firms' decisions by calculating the efficient level of legal sanctions (S_{t+1}^*) that dissuades "type k = 1" firms from choosing continuation at time t: indeed, derived function (16) becomes negative and $p(C_t|1)^*$ reduces to zero when:

$$S_{t+1}^{*} > C_{t+1}^{F} + \frac{1}{(1 - p(x|1))} \cdot \left(p(x|1) \cdot \left(x \cdot I_{t+1}^{c} + V_{t+1}^{L} - (1 + i_{t-\epsilon}) \cdot D_{t-\epsilon} \right) - C_{t}^{F} \right)$$

$$> f_{0} \left(p(x|1); x; I_{t+1}^{c}; V_{t+1}^{L}; C_{t+1}^{F}; C_{t}^{F}; D_{t-\epsilon}; i_{t-\epsilon} \right)$$
(17)

f(.) is the minimum level of legal sanctions that makes the continuation of bad firms unattractive.

B. *The "type* k = 0" *firms* ' *programs*

Profitable firms make a similar calculus. Nevertheless, they do not recover anything in the case of financial distress (see above, table 1: case $p(0|C_t;x_{t+1}=0) < \hat{p}_0$). Their programs can be written as follows:

$$\underset{\left(p(C_{t}|0)\right)}{\operatorname{Max}} \operatorname{E}_{t}\left(\Pi|0\right) = p(C_{t}|0) \cdot p(x|0) \cdot \left(x \cdot I_{t+1}^{c} + V_{t+1}^{L} - (1 + i_{t-\epsilon}) \cdot D_{t-\epsilon}\right) + (1 - p(C_{t}|0)) \cdot C_{t}^{F} \quad (18)$$

PROPOSITION 5. The legislator cannot define a level of legal sanctions that incites profitable firms to continue their activity.

Proof. The resolution of program (18) leads to a relation which is independent of the level of legal sanctions:

$$\frac{dE_t(\Pi|0)}{dp(C_t|0)} = p(x|0) \cdot \left(x \cdot I_{t+1}^c + V_{t+1}^L - (1 + i_{t-\epsilon}) \cdot D_{t-\epsilon}\right) - C_t^F$$
(19)

Whereas the legislator can enforce constraint (17), the choices of profitable firms are beyond his control: for any level of legal sanctions, derived function (19) may be negative and, then, $p(C_t|0)^*$ may equal zero. In that case, all firms voluntarily liquidate, regardless of their types.

Hence, the legislator's incentive power is restricted to the choices of non-profitable firms and leads to their voluntary liquidation at time t (if and only if the level of legal sanctions strictly exceeds $f_0(.)$).

PROPOSITION 6. When the legislator dissuades bad firms from continuing, condition " $p(0|C_t;x_{t+1}=0) < \hat{p}_0$ " cannot hold anymore.

Proof. There are two possible cases when the legislator enforces condition (17):

- <u>expression (19) is positive</u>: only good firms continue; $p(0|C_t;x_{t+1}=0)$ is then equal to one and condition " $1 < \hat{p}_0$ " is impossible;
- <u>expression (19) is negative</u>: $p(0|C_t;x_{t+1}=0)$ is not defined because no firm chooses continuation, and the considered case cannot be verified.

III.1.2. High proportion of profitable firms: $p(0|C_t:x_{t+1}=0) \ge \hat{p}_0$

Here, in case of financial distress, firms recover identical amounts, whatever their types: the only difference between expected profits derives from the probabilities of success of the investment project: p(x|k). Besides, an essential point to be stressed is that conditional probability $p(0|C_{t},x_{t+1}=0)$ appears in the expression of expected profits (see above: table 1, pooling equilibrium). This probability is calculated by the bank at time t+1, *when continuation is observed*: learning continuation at time t+1, the bank can update its beliefs about the type of the firm, using Bayes' theorem: Bayesian updating leads to posterior probabilities of being a "type (*k*)" firm. In particular, for k = 0:

$$p(0|C_{t}; x_{t+1} = 0) = \frac{p_{0} \cdot p(C_{t}|0) \cdot (1 - p(x|0))}{p_{0} \cdot p(C_{t}|0) \cdot (1 - p(x|0)) + (1 - p_{0}) \cdot p(C_{t}|1) \cdot (1 - p(x|1))}$$
(20)

At equilibrium, $p(0|C_t;x_{t+1}=0)$ is replaced by this expression into the firms' programs. As usual, "type (*k*)" firms use probabilities $p(C_t|k)$ to maximize their expected profits.

A. <u>The "type k = 1" firms' programs</u>

The resolution method is similar to the one described under separating equilibrium. However, in case of failure, firms earn $C_{t+1}^{F} - (1 - p(0|C_{t};x_{t+1}=0)) \cdot S_{t+1}$ (see above, table 1) and their maximization programs become:

$$\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l))}{\underset{(p(C_{t}|l)}{\underset{(p(C_{t}|l)}{\underset{(p(C_{t}|l)}{\underset{(p(C_{t}|l)}{\underset{(p(C_{t}|l)}{\underset{(p(C_{t}|l)}{\underset{(p(C_{t}|l)}{\underset{(p(C_{t}|l)}{\underset{(p(C_{t}|l)}{\underset{(p(C_{t}|l)}{\underset{(p(C_{t}|l)}{\underset{(p(C_{t}|l)}{\underset{(p(C_{t}|l)}{\underset{(p(C_{t}|l)}{\underset{(p(C_{t}|l)}{\underset{(p(C_{t}|l)}{\underset{(p(C_{t}|l)}{\underset{(p(C_{t}|l)}{\underset{(p(C_{t}|l)}{\underset{(p(C_{t}|l)}{\underset{(p(C_{t}|l)}{\underset{(p(C_{t}|l)}{\underset{(p(C_{t}|l)}{\underset{(p(C_{t}|l)}{\underset{(p(C_{t}|l)}{\underset{(p(C_{t}|l)}{\underset{(p(C_{t}|l)}{\underset{(p(C_{t}|l)}{\underset{(p(C_{t}|l)}{\underset{(p(C_{$$

PROPOSITION 7. At equilibrium, the level of legal sanctions and the probability of continuation for good firms affect the probability of continuation for bad firms:

$$p(C_{t}|1)^{*} = p(C_{t}|0) \cdot \frac{p_{0} \cdot (1 - p(x|0))}{(1 - p_{0}) \cdot (1 - p(x|1))} \cdot \#(S_{t+1})$$
(22)

with
$$\#(S_{t+1}) = \sqrt{\frac{(1 - p(x|1)) \cdot S_{t+1}}{(1 - p(x|1)) \cdot S_{t+1} - \#(1)}} - 1$$
 and
 $\#(1) = p(x|1) \cdot \begin{pmatrix} x \cdot I_{t+1}^c + V_{t+1}^L \\ -(1 + i_{t-\epsilon}) \cdot D_{t-\epsilon} \end{pmatrix} + (1 - p(x|1)) \cdot C_{t+1}^F - C_t^F$

It is important to notice that $\#(S_{t+1})$ nears zero when S_{t+1} moves towards infinite.

Proof. Using relation (20) and notations $\#(S_{t+1})$ and #(1), the resolution of program (21) leads to the following equation:

$$\frac{\partial E_{t}(\Pi|l)}{\partial p(C_{t}|l)} = 0 \iff p(C_{t}|l)^{2} + p(C_{t}|l) \cdot \left(\frac{2p_{0} \cdot (1 - p(x|0)) \cdot p(C_{t}|0)}{(1 - p_{0}) \cdot (1 - p(x|1))}\right) + \frac{\#(1) \cdot (p_{0} \cdot (1 - p(x|0)) \cdot p(C_{t}|0))^{2}}{(\#(1) - (1 - p(x|1)) \cdot S_{t+1}) \cdot ((1 - p_{0}) \cdot (1 - p(x|1)))^{2}} = 0$$
(23)

Equation (23) has two real solutions provided its discriminant is positive : this is the case when $\#(1) \le (1 - p(x|1)) \cdot S_{t+1}$. Only one of these solutions is positive²⁹ and is equal to:

$$p(C_{t}|1)^{*} = p(C_{t}|0) \cdot \frac{p_{0} \cdot (1 - p(x|0))}{(1 - p_{0}) \cdot (1 - p(x|1))} \cdot \#(S_{t+1})$$
(24)

The legislator can influence the decisions of bad firms. But these are also influenced by the behavior of profitable firms. \blacksquare

Besides, probability $p(C_t|1)^*$ must be defined between zero and one. This condition implies the two following inequalities and characterizes a definition interval for $p(C_t|1)^*$:

$$p(C_{t}|1)^{*} \ge 0 \implies \#(1) \ge 0$$

$$p(C_{t}|1)^{*} \le 1 \implies \#(1) \le (1 - p(x|1)) \cdot S_{t+1}$$

$$\times \left(1 - \left(\frac{p_{0} \cdot (1 - p(x|0)) \cdot p(C_{t}|0)}{p_{0} \cdot (1 - p(x|0)) \cdot p(C_{t}|0) + (1 - p_{0}) \cdot (1 - p(x|1))} \right)^{2} \right)$$
(26)

²⁹ The associated profit is maximum. Indeed, remembering that legal sanctions are always positive, we have: $\frac{\partial^2 E_t(\Pi|l)}{\partial p(C_t|l)^2} = -\frac{2p_0^2 (1-p(x|0))^2 p(C_t|0)^2 (1-p_0) (1-p(x|l))^2 S_{t+1}}{\left[p_0 (1-p(x|0)) p(C_t|0) + (1-p_0) (1-p(x|l)) p(C_t|l)\right]^3} \le 0$

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B. <u>The "type k = 0" firms programs</u>

"Type k = 0" firms decide continuation with probability $p(C_t|0)$. Corresponding gains are similar: the only difference lies in the probability of success of the investment project, which is higher for profitable firms. The "type k = 0" firms' programs are the following:

$$\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0))}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset{(p(C_{t}|0)}{\underset$$

PROPOSITION 8. At equilibrium, the level of legal sanctions and the probability of continuation for bad firms affect the probability of continuation for good firms. However, unlike proposition 7, the optimal level of $p(C_t|0)$ is either equal to zero or equal to one.

Proof. Using relation (20) and program (27), we have:

$$\frac{\partial E_{t}(\Pi|0)}{\partial p(C_{t}|0)} = 0 \quad \Leftrightarrow \quad \#(0) - (1 - p(x|0)) \\ \times \left(\frac{(1 - p_{0}) \cdot (1 - p(x|1)) \cdot p(C_{t}|1)}{p_{0} \cdot (1 - p(x|0)) \cdot p(C_{t}|0) + (1 - p_{0}) \cdot (1 - p(x|1)) \cdot p(C_{t}|1)}\right)^{2} \cdot S_{t+1} = 0$$
(28)

with
$$\#(0) = p(x|0) \cdot \begin{pmatrix} x \cdot I_{t+1}^{c} + V_{t+1}^{L} \\ -(1 + i_{t-\epsilon}) \cdot D_{t-\epsilon} \end{pmatrix} + (1 - p(x|0)) \cdot C_{t+1}^{F} - C_{t}^{F}$$

Equation (28) has one positive solution. Nevertheless, it cannot be accepted, because the associated profit is minimum: function $E_t(\Pi|0)$ is convex³⁰. Because of this convexity, each "type k = 0" firm compares the two extreme solutions, $p(C_t|0)^* = 1$ and $p(C_t|0)^* = 0$, choosing the most profitable one. The first solution $(p(C_t|0)^* = 1)$ is chosen if and only if the associated profit is greater than it would be with the other possible choice $(p(C_t|0)^* = 0)$. This implies:

$$\begin{cases} If \quad p(x|0) \cdot \left(x \cdot I_{t+1}^{c} + V_{t+1}^{L} - (1 + i_{t-\epsilon}) \cdot D_{t-\epsilon} \right) + (1 - p(x|0)) \\ \times \left(C_{t+1}^{F} - \left(1 - \frac{p_{0} \cdot (1 - p(x|0))}{p_{0} \cdot (1 - p(x|0)) + (1 - p_{0}) \cdot (1 - p(x|1)) \cdot p(C_{t}|1)} \right) \cdot S_{t+1} \right) \ge C_{t}^{F} \quad (29) \\ \Rightarrow \quad p(C_{t}|0) = 1 \\ Else \quad \Rightarrow \quad p(C_{t}|0) = 0 \end{cases}$$

³⁰ Indeed :
$$\frac{\partial^{2} E_{t}(\Pi|0)}{\partial p(C_{t}|0)^{2}} = \underbrace{\frac{2p_{0} \cdot (1-p(x|0))^{2} \cdot (1-p_{0})^{2} \cdot (1-p(x|1))^{2} \cdot p(C_{t}|1)^{2} \cdot S_{t+1}}{\left[p_{0} \cdot p(C_{t}|0) \cdot (1-p(x|0)) + (1-p_{0}) \cdot p(C_{t}|1) \cdot (1-p(x|1))\right]^{3}}_{\geq 0}}$$

At equilibrium, $p(C_t|1)$ can be replaced by $p(C_t|1)^*$ and decision rule (29) is rewritten as follows (reusing notation #(0)):

$$\begin{cases} \text{If } \#(0) - (1 - p(x|0)) \cdot \frac{\#(S_{t+1})}{(\#(S_{t+1}) + 1)} \cdot S_{t+1} \ge 0 \\ \Rightarrow \begin{cases} p(C_t|0)^* = 1 \\ p(C_t|1)^* = \frac{p_0 \cdot (1 - p(x|0))}{(1 - p_0) \cdot (1 - p(x|1))} \cdot \#(S_{t+1}) \end{cases}$$
(30)
Else $\Rightarrow p(C_t|0)^* = p(C_t|1)^* = 0$

The level of legal sanctions (S_{t+1}) influences the probabilities of continuation and liquidation. Taking that point into account, the legislator has an incentive power, which can be used to select firms in an optimal way.

C. The legislator's objective

Referring to the firms' programs described above, the legislator knows there are two pairs of probabilities, which are defined at equilibrium:

(0,0) and
$$\left(1; \frac{p_0(1-p(x|0))}{(1-p_0)(1-p(x|1))} \cdot \#(S_{t+1})\right).$$

A way of specifying the legislator's objective function (noted $E_t(\Pi^L)$) is to consider that he maximizes the social surplus (which is equivalent to maximizing the value of the firm). The social surplus totals the bank's expected profit (noted $E_t(\Pi^B)$), profitable firms' expected profits ($E_t(\Pi|0)$), and non-profitable firms' expected profits ($E_t(\Pi|1)$). Normalizing to one the total population of firms, there are p_0 "type k = 0" firms and $1 - p_0$ "type k = 1" firms in the economy:

$$\max_{(\mathbf{S}_{t+1})} \mathbf{E}_t \left(\boldsymbol{\Pi}^L \right) = \mathbf{p}_0 \cdot \mathbf{E}_t \left(\boldsymbol{\Pi} | \mathbf{0} \right) + (1 - \mathbf{p}_0) \cdot \mathbf{E}_t \left(\boldsymbol{\Pi} | \mathbf{1} \right) + \mathbf{E}_t \left(\boldsymbol{\Pi}^B \right)$$

with:

$$E_{t}(\Pi^{B}) = p_{0} \cdot \begin{pmatrix} p(C_{t}|0) \cdot \begin{pmatrix} p(x|0) \cdot (1 + i_{t-\varepsilon}) \cdot D_{t-\varepsilon} + (1 - p(x|0)) \\ \times (V_{t+1}^{L} - C_{t+1}^{F} + (1 - p(0|C_{t}; x_{t+1} = 0)) \cdot S_{t+1}) \end{pmatrix} \\ + (1 - p(C_{t}|0)) \cdot (V_{t}^{L} - C_{t}^{F}) \end{pmatrix}$$

$$+ (1 - p_{0}) \cdot \begin{pmatrix} p(C_{t}|1) \cdot \begin{pmatrix} p(x|1) \cdot (1 + i_{t-\varepsilon}) \cdot D_{t-\varepsilon} + (1 - p(x|1)) \\ \times (V_{t+1}^{L} - C_{t+1}^{F} + (1 - p(0|C_{t}; x_{t+1} = 0)) \cdot S_{t+1}) \end{pmatrix} \\ + (1 - p(C_{t}|1)) \cdot (V_{t}^{L} - C_{t}^{F}) \end{pmatrix}$$

$$- (1 + i_{t-\varepsilon}^{B}) \cdot D_{t-\varepsilon}$$

$$(31)$$

where $i_{t-\epsilon}^{B}$ is the refinancing rate at time $(t-\epsilon)$.

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PROPOSITION 9. The efficient level of legal sanctions is infinite. However, because legal sanctions are bound to the level of the assets shortage (see section I), the legislator's action only leads to a second best optimum, which is defined by S_{t+1}^{M} .

Proof. In equation (31), terms relative to the profit sharing between lenders and borrowers compensate each other, so that only the project's *economic* value remains. From an economic point of view, in case of continuation, the project yields the certain amount V_{t+1}^{L} plus $x \cdot I_{t+1}^{c}$ with probability p(x|k). In case of liquidation, the economic value of the project is V_{t}^{L} . Equation (31) can be rewritten in order to underscore these amounts:

$$\begin{aligned} \underset{(S_{t+1})}{\text{Max}} & E_{t} \left(\Pi^{L} \right) = p_{0} \cdot \left(p(C_{t}|0) \cdot \left(V_{t+1}^{L} + p(x|0) \cdot x \cdot I_{t+1}^{c} \right) + \left(1 - p(C_{t}|0) \right) \cdot V_{t}^{L} \right) \\ & + (1 - p_{0}) \cdot \left(p(C_{t}|1) \cdot \left(V_{t+1}^{L} + p(x|1) \cdot x \cdot I_{t+1}^{c} \right) + \left(1 - p(C_{t}|1) \right) \cdot V_{t}^{L} \right) \\ & - \left(1 + i_{t-\epsilon}^{B} \right) \cdot D_{t-\epsilon} \end{aligned}$$
(32)

Equation (32) depends on the type of firms, their proportion in the economy, and the probabilities of continuation and liquidation³¹. Replacing $p(C_t|1)$ by its equilibrium value, in function of $p(C_t|0)$ and $\#(S_{t+1})$, equation (32) can be rewritten as follows:

$$\begin{split} & \underset{(S_{t+1})}{\text{Max}} \mathbb{E}_{t} \left(\Pi^{L} \right) = \begin{pmatrix} V_{t}^{L} - \left(l + i \frac{B}{t-\epsilon} \right) \\ & \times D_{t-\epsilon} \end{pmatrix} \\ & + p_{0} \cdot p(C_{t}|0) \cdot \left[\underbrace{\frac{V_{t+1}^{L} + p(x|0) \cdot x \cdot I_{t+1}^{c} - V_{t}^{L}}_{\geq 0}}_{+ \#(S_{t+1}) \cdot \underbrace{\left(V_{t+1}^{L} + p(x|1) \cdot x \cdot I_{t+1}^{c} - V_{t}^{L} \right)}_{<0} \right]$$
(33)

Hence, the legislator's objective function corresponds to the equation of a straight line, whose *y*-axis intercept is equal to $V_t^L - (1 + i_{t-\epsilon}^B) \cdot D_{t-\epsilon}$. Of course, the legislator would like to obtain a greater level than this minimal value. He can do so by modifying the value of S_{t+1} , in order to [a] maximize the term in square brackets and [b] incite $p(C_t|0)$ to be equal to one. This is possible when S_{t+1} moves towards infinite:

- Firstly, when S_{t+1} moves towards infinite, $\#(S_{t+1})$ nears zero and the second term between square brackets disappears (simultaneously, it may be pointed out that an infinite level value of legal sanctions induces a zero value of probability $p(C_t|1)$);

³¹ If the legislator could directly control the level of probabilities $p(C_t|k)$ (which is of course impossible), the objective function would be maximized when $p(C_t|0)=1$ and $p(C_t|1)=0...$ This situation corresponds to our implicit initial objectives: continuation for "k=0" firms and liquidation for "type k=1" ones.

- Secondly, looking at equation (30), we know that $p(C_t|0)$ equals one if we have:

$$\begin{array}{l}
\#(0) - (1 - p(x|0)) \cdot \frac{\#(S_{t+1})}{(\#(S_{t+1}) + 1)} \cdot S_{t+1} \ge 0 \\
\Leftrightarrow \underbrace{S_{t+1} - \sqrt{S_{t+1}} \cdot \left(S_{t+1} - \frac{\#(1)}{(1 - p(x|1))}\right)}_{\underbrace{S_{t+1} \to +\infty} 0} \le \frac{\#(0)}{(1 - p(x|0))} \end{array} \tag{34}$$

When S_{t+1} is infinite, inequality (34) is always verified³². Indeed, because #(1) is supposed to be positive³³, #(0) is positive too (#(0) \geq #(1)). Hence, by choosing an infinite level of S_{t+1} , the legislator maximizes the social surplus and incites profitable firms to continue ($p(C_t|0)=1$), and non-profitable ones to liquidate ($p(C_t|1) \rightarrow 0$).

III.1.3. Legislator's intervention under the "severe legal system": a synthesis

Table 2 summarizes all discussed cases and corresponding results:

Possible cases		Is the case optimal?	Is the case accepted?	Optimal value of S_{t+1}^{*}	Firms' decision
$p(0 C_t) < \hat{p}_0$	Expression $(19) \ge 0$	Yes	No ^(**)	$S_{t+1}^* > f_0(.)$	$p(C_t 0)^* = 1$ $p(C_t 1)^* = 0$
	Expression (19) < 0	No	Uncertain	Any (no effect)	$p(C_t 0)^* = 0$ $p(C_t 1)^* = 0$
$p(0 C_t) \ge \hat{p}_0$	$\#(0) > \#(1) \ge 0$	$\begin{array}{c} \operatorname{Yes}(\Rightarrow\\ \operatorname{second} \operatorname{best}) \end{array}$	Yes ^(****)	Infinite (\Rightarrow bound to S_{t+1}^{M})	$p(C_t 0)^* = 1$ $p(C_t 1)^* \to 0$
	$\#(0) \ge 0 > \#(1)$		No	Any	$p(C_t k)^*$ is not defined $(\forall k)^{(***)}$
	0 >#(0) >#(1)	No	Uncertain	Any (no effect)	$p(C_t 0)^* = 0$ $p(C_t 1)^* = 0$

Table 2: Legal sanctions at equilibrium, under the "severe legal system" (*)

(*) With:
$$\begin{cases} \#(0) = p(x|0) (x \cdot I_{t+1}^{c} + V_{t+1}^{L} - (1 + i_{t-\epsilon}) \cdot D_{t-\epsilon}) + (1 - p(x|0)) C_{t+1}^{F} - C_{t}^{F} \\ \#(1) = p(x|1) (x \cdot I_{t+1}^{c} + V_{t+1}^{L} - (1 + i_{t-\epsilon}) \cdot D_{t-\epsilon}) + (1 - p(x|1)) C_{t+1}^{F} - C_{t}^{F} \end{cases}$$

(**) This case does not hold anymore when condition $S_{t+1}^* > f_0(.)$ prevails.

(***) When #(1) < 0, $p(C_t|1)$ is also negative, which is impossible.

(****) While S_{t+1} is increasing, the case $p(0|C_t) < \hat{p}_0$ becomes less likely and costly formal bankruptcies are all the less frequent. As a matter of fact, when S_{t+1} is infinite, we have (see below):

³² Because of the convergence speed of $\#(S_{t+1})$, which is the greatest one.

³³ Otherwise, $p(C_t|1)$ would not be a probability: see above (condition (25)).

$$\begin{cases} p(0|C_{t})^{*} = \frac{p_{0} \cdot p(C_{t}|0)^{*}}{p_{0} \cdot p(C_{t}|0)^{*} + (1-p_{0}) \cdot p(C_{t}|1)^{*}} & \xrightarrow{S_{t+1} \to +\infty} 1 \\ \hat{p}_{0} = \frac{S_{t+1} - C_{t+1}^{F}}{S_{t+1}} & \xrightarrow{S_{t+1} \to +\infty} 1 \end{cases}$$

Let's show that while each of these probabilities is getting closer to one, $p(0|C_t)$ remains greater than \hat{p}_0 , so that informal renegotiation is always decided:

$$p(0|C_{t})^{?} \stackrel{\stackrel{?}{\Rightarrow}}{p_{0}} \stackrel{\Leftrightarrow}{\underset{(p|C_{t}|0)=1}{\longrightarrow}} \frac{p_{0}}{p_{0}+(1-p_{0})\cdot\left(\frac{\#(S_{t+1})}{\#(S_{t+1})+1}\right)} \geq \frac{S_{t+1}-C_{t+1}^{F}}{S_{t+1}}$$

$$\Leftrightarrow \quad (1-p_{0})\cdot\left(1-\sqrt{\frac{(1-p(x|1))S_{t+1}-\#(1)}{(1-p(x|1))\cdot S_{t+1}}}\right)\cdot(S_{t+1}-C_{t+1}^{F}) - p_{0} \cdot C_{t+1}^{F} \leq 0$$

$$\underbrace{(1-p_{0})\cdot\left(1-\sqrt{\frac{(1-p(x|1))S_{t+1}-\#(1)}{S_{t+1}\rightarrow +\infty}}\right)}_{\overrightarrow{S_{t+1}\rightarrow +\infty} \rightarrow 0}$$

This inequality is always verified and the legislator's intervention implies the disappearance of costly formal bankruptcies.

Under bayesian updatings, an infinite level of legal sanctions maximizes the social surplus and incites profitable firms to continue and non-profitable ones to liquidate. It is important to stress that it is a credible threat: at equilibrium, legal sanctions are never applied, because their dissuasive power eliminates the risk of continuation from bad projects. Simultaneously, modifying the population's structure (from now on, quasi-exclusively made up of good firms), the legislator eliminates asymmetric information at time t+1 and reduces the occurrence of formal bankruptcy procedures. This confirms Haugen and Senbet's result (1978 and 1988), according to which any bankruptcy procedure can be avoided, by internalizing bankruptcy costs.

Nevertheless, the legislator's interference only attains a second best optimum because S_{t+1} cannot exceed S_{t+1}^{M} . Moreover, the legislator's intervention does not always lead to optimality. This is the case when #(0) and #(1) are both negative: here, the legislator cannot prevent the premature liquidation of all projects. This case is more likely when bankruptcy costs get higher (cf. parameter β_F , which affects C_t^F and C_{t+1}^F), because this reduces the values of #(0) and #(1). Thus, we obtain the following paradoxal result: too high a level of bankruptcy costs prevents the law from reaching optimality. Yet, this result must be moderated, observing that the legislator plays an important role in the determination of bankruptcy costs.

³⁴ Moreover, some simplifying assumptions should be tempered (the manager's personal possessions are not infinite, firms may uncertainly know their own type), which might move the economy away from optimality.

III.2. "Moderate legal system": $S_{t+1} < C_{t+1}^F$

PROPOSITION 10. The "moderate legal system" is unstable because of the legislator's intervention.

Proof. The "moderate legal system" implies the same payments as for the "severe legal system" when $p(0|C_t) \ge \hat{p}_0$. Agents behave in the same way as before, because condition $S_{t+1} < C_{t+1}^F$ has never been required for the demonstration.

Nevertheless, in that case, we know that the legislator's intervention leads to an infinite level of legal sanctions, which leads to the "severe legal system". Hence, equilibrium under the "moderate legal system" is unstable when firms use mixed strategies with Bayesian updating of the bank's beliefs.

CONCLUDING REMARKS

Our model analyzes the conditions under which legal sanctions allow an efficient selection of heterogeneous investment projects. The considered bankruptcy code is a *"primitive creditor system"* and financial distress leads to an arbitration between private agreement and formal bankruptcy.

We find that it is not always possible to define a collectively optimal level of legal sanctions: in that case, all projects are prematurely liquidated, whatever their profitability. This situation is all the more likely to happen with [a] low firms' profitability p(x|k), [b] little return per unit of capital (x), [c] unfavourable assets resale conditions (ρ), [d] high interest rates ($i_{t-\epsilon}$), and [e] excessive bankruptcy costs (β_F). On the latter point, an interesting opposition appears between the legislator's initial objectives (the discrimination between bad and good investment projects) and legal operating expenses (bankruptcy costs). An inexpensive legal system is all the more likely to be collectively efficient.

In other cases, the legislator is able to define a collectively optimal level of legal sanctions that incites good projects to continue and bad ones to liquidate. This interference on the part of the legislator implies the generalization of private agreements in case of financial distress. The optimal level of legal sanctions must move towards infinite. Nevertheless, because legal sanctions are bound to the level of the assets shortage, the legislator's action only leads to a second best optimum.

Of course, these results should be nuanced by improving the considered assumptions. In particular, the manager's personal possessions may not be infinite. Besides, we have supposed that each manager perfectly knows his type, which is not always the case in practice. Some managers may sincerely pursue a non-profitable affair: here, the tribunal's appreciation is crucial.

In Europe, delays in filing are numerous. Especially, in France, more than 80% of distressed firms are liquidated immediately³⁵, which indicates that many firms pursue a non-profitable business until it is too late: the main problem is prevention. Our model suggests some economic elements that could be taken into account by lawmakers: debtor-oriented legislations should be characterized by a more severe application of legal sanctions in order to dissuade from sub-optimal continuation. This would allow a better selection of investment projects and would lead to cheaper private agreements. Nevertheless, we have shown that this dissuasion power does not always operate. Statistically, French tribunals are relatively clement toward distressed managers. As a matter of fact, Blazy and Combier (1997) show that, whereas 27% of immediately liquidated firms have management problems³⁶ before bankruptcy, only 4% are sanctioned. This rate increases for criminal sanctions (23%) but such sanctions do not induce financial transfer from managers to creditors.

BIBLIOGRAPHY

- Aghion P., La Porta Drago R., Lopez-de-Silanes F., and Moore J. (1997). A New Bankruptcy Procedure That Uses Multiple Auctions, *European Economic Review*, 41, 461-473.
- Aghion P., O. Hart, and J. Moore (1992). The Economics of Bankruptcy Reform, *Journal* of Law, Economics and Organization, 8, n°3, 523-546.
- Aghion P. and P. Bolton (1992). An Incomplete Contracts Approach to Financial Contracting, *Review of Economic Studies*, 59, 473-494.
- Baxter N.D. (1967). Leverage, Risk of Ruin and the Cost of Capital, *Journal of Finance*, 22, 395-403.
- **Bebchuk L.A. (1988).** A New Approach to Corporate Reorganizations, *Harvard Law Review*, 101, 775-804.
- Berkovitch E. and R. Israel (1999). Optimal Bankruptcy Laws Accross Different Economic Systems, *Review of Financial Studies*, 12, n°2, 347-377.
- **Bierman H.S. and Fernandez L. (1998).** *Game Theory* (2nd edition), Addison-Wesley Ed., United States.
- **Blazy R. (1999).** Modifications de comportement précédant la défaillance d'entreprise : l'impact des cessions d'actifs sur la relation de crédit, *Louvain Economic Review*, 65, n°2.
- **Blazy R. and J. Combier (1997).** La défaillance d'entreprise : causes économiques, traitement judiciaire et impact financier, collection I.N.S.E.E. Méthodes, n°.72-73, Economica.
- Drukarczyk J. (1991). Secured Debt, Bankruptcy, and the Creditors' Bargain Model, *International Review of Law and Economics*, 11, 203-221.

³⁵ From this point of view, we are especially interested in the impact of legal sanctions on the bad firms' liquidation choice.

³⁶ This title includes the following causes of bankruptcy (most of them may justify legal sanctions): "inefficient accountancy", "lack of knowledge on the part of managers", "disagreement among managers", "excessive financial withdrawal", "misappreciation of cost prices", "firm's transmission difficulties", "bad valuation of stocks", and "insufficient provisions".

- Franks J.R. and K.G. Nyborg (1992). Workouts Versus Formal Bankruptcy: Incentives and Inefficiencies Under Different Bankruptcy Codes, *London Business School*, working paper, March.
- **Franks J.R. and K.G. Nyborg (1996).** Control Rights, Debt Structure, and the Loss of Private Benefits: The Case of the U.K. Insolvency Code, Review of Financial Studies, 9, n°4, 1165-1210.
- Haugen R.A. and L.W. Senbet (1978). The Insignificance of Bankruptcy Costs to the Theory of Optimal Capital Structure, *Journal of Finance*, 33, n°2, 383-393.
- **Haugen R.A. and L.W. Senbet (1988).** Bankruptcy and Agency Costs: Their Significance to the Theory of Optimal Capital Structure, *Journal of Financial and Quantitative Analysis*, 23, n°1, 27-38.
- Jensen M.C. and W.H. Meckling (1976). Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure, *Journal of Financial Economics*, 3, 305-360.
- Kim J., K. Ramaswamy, and S. Sundaresan (1993). Does Default Risk in Coupons Affect the Valuation of Corporate Bonds? A Contingent Claims Model, *Financial Management*, autumn, 117-131.
- Scott J. (1976). A Theory of Optimal Capital Structure, Bell Journal of Economics, 7, 33-54.
- Webb D.C. (1987). The Importance of Incomplete Information in Explaining the Existence of Costly Bankruptcy, *Economica*, 54, 279-288.
- White M.J. (1989). The Corporate Bankruptcy Decision, *Journal of Economic Perspectives*, 3, n°2, 129-151.
- White M.J. (1996). The Costs of Corporate Bankruptcy: A U.S.-European Comparison, *in Corporate Bankrupty Economic and Legal Perspectives*, J.S. Bhandari and L.A. Weiss Ed., Cambridge University Press.



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