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WHICH OPTIMAL DESIGN FOR LLDA?

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Abstract

Lottery-linked deposit accounts are financial assets that provide an interest rate determined by a lottery. The aim of this study is to determine the optimal design of these financial assets (under cumulative prospect theory (CPT) framework). We underline that the weighting functions usually specified in the literature should be re-modeled if we want to apply CPT to finance. We propose to replace them by another functional form that preserves the main characteristics of the inverse S-shape specification, but whose slope at zero is finite. The optimal structure of payments obtained is consistent with the conclusions of behavioral portfolio theory (2000).

Keywords : Lottery-Linked-Deposit Account, Cumulative Prospect Theory, Design optimal, Probability Weighting.

JEL Classification : D81, G11, C01.

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1 Introduction

Expected utility theory has been considered for several decades to be a benchmark for describing decision making under risk. According to this normative model of rational choices, attitude towards risk is entirely characterized by the shape of the utility function. In economics and finance, it is generally assumed that investors are risk-averse. Their behavior is then modeled by a concave utility function. However, Pfiffelmann and Roger (2005) and Pfiffelmann (2007b) point out that the popularity of some financial assets, such as lottery-linked-deposit-accounts (LLDA), challenges this assumption. LLDA are financial assets that provide an interest rate determined by a lottery (Guillen and Tschoegl, 2002). Their existence cannot be explained in the framework of expected utility models since a risk-averse investor would accordingly always prefer to get the expected value of a lottery rather than participate in the gamble. Pfiffelmann and Roger (2005) and Pfiffelmann (2007b) show that rank dependent expected utility (Quiggin, 1982) and cumulative prospect theory (CPT) (Tversky and Kahneman, 1992) provide a good explanation for the emergence of these deposit accounts by integrating simultaneous risk-averse and risk-seeking behaviors. By comparing two LLDA, they establish that a modification, "ceteris parebus", of the structure of payments of the lottery associated to these kinds of assets, in the sense of an increase in the positive asymmetry, could improve the appeal of the account. Therefore, a link exists between the design of these financial assets, their popularity and the cost of the issuer.

The purpose of the present paper is to use this relation to determine the optimal design of LLDA. We investigate on the "best" structure of payments of these types of financial assets. In order to optimize the lottery design, we maximize the satisfaction of investors given an issuer's cost equal to the risk-free interest rate. The optimization program leads to an optimal and relevant structure of payments. The results show that the lottery should be strongly asymmetrical. The explanation lies in the tendency individuals have to overweight the extremely low probability of the desired outcome. Links with the first version (the single account version) of the behavioral portfolio theory (BPT- SA) developed by Shefrin and Statman (2000) can thus be established. In a second step, we investigate on the optimal design of LLDA when both payments and probabilities should be determined; we look for the optimal structure of payments and the associated optimal probabilities. Our results allow us to discuss the shape of the Tversky and Kahneman's weighting function. We underline that there are situations for which the subjective value of the jackpot of the lottery weighted by its decision weight can be infinitely high. This solution suggests that an infinite jackpot associated with an extremely small probability should be sufficient to attract lots of investors. This is quite unrealistic. Therefore, if we want to apply CPT to portfolio selection or to any fields of finance, this theory has to be re-modeled (De Giorgi and Hens). Actually, we obtain such a result because the slope of the weighting function at zero is infinite. The more the probabilities are low, the more they are overweighted. We propose to replace the specification of the weighting function proposed by Tversky and Kahneman with a polynomial functional. This functional form exhibits a finite slope at the origin and preserves the main characteristics of the Tversky and Kahneman's inverse S-shape specification: overweighting of small probabilities, underweighting of moderate and high probabilities and the decreasing sensibility principle We investigate on the optimal design of LLDA in this new framework. Our results are consistent with the multiple accounts version of the behavioral portfolio theory (BPT-MA)

The paper is structured as follows: In section II, we review the main characteristics of CPT developed by Tversky and Kahneman. Section III establishes the optimal design of LLDA in CPT framework. In section IV, we modify the specification of the weighting function and analyze the new structure of payments obtained with this functional. The paper concludes in section V with a summary of our findings.

2 Review of Prospect Theory

In this section, we introduce CPT by first reviewing the main observations established by Tversky and Kahneman.

2.1 Main observations

- As in expected utility theory, investors determine the subjective value of each outcome via a value function. However, under CPT utility is defined over gains and losses rather than over final asset position, so risky prospects are evaluated relatively to a reference point. This reference point corresponds to the asset position one expects to reach.
- Investors transform probabilities via a weighting function. They overweight the probabilities of extreme outcomes (events at the tails of the distribution) and underweight outcomes with average probabilities. The weighting function is concave near 0 and convex near 1.
- The sensibility relatively to the reference point is decreasing. The value function v is then concave over gains and convex over losses
- Investors dislike loss. The loss of \$100 creates a distress greater than the satisfaction generated by the gain of the same amount of money. The value function is then steeper for losses than for gains.
- Experimental evidence has established a “fourfold pattern of risk attitudes”: risk aversion for most gains and low probability losses, and risk seeking for most losses and low probability gains.

2.2 Modelisation.

Consider a prospect X defined by:

$$X = ((x_i, p_i) i = -m, ...n)$$

with $x_{-m} < x_{-m+1} < .... < x_0 = 0 < x_1 < x_2 < ... < x_n$.

Gains and losses are evaluated differently by investors. The subjective utility V of a prospect X is then defined by:

$$V(X) = V(X^{+}) + V(X^{-}) \qquad (1)$$

where $X^{+} = \max(X;0)$ et $X^{-} = \min(X;0)$.

We set:

$$\begin{aligned} V(X^{+}) &= \sum_{i=0}^{n} \pi_i^{+} v(x_i) \\ V(X^{-}) &= \sum_{i=-m}^{0} \pi_i^{-} v(x_i) \end{aligned} \qquad (2)$$

where v is a strictly increasing value function defined with respect to a reference point satisfying $v(x_0) = v(0) = 0$.

$\pi^{+} = (\pi_0^{+}, ...\pi_n^{+})$ and $\pi^{-} = (\pi_{-m}^{-}, , ...\pi_0^{-})$ are the weighting functions for gains and losses respectively defined by:

$$\begin{aligned} \pi_n^{+} &= w^{+}(p_n) \\ \pi_{-m}^{-} &= w^{-}(p_{-m}) \\ \pi_i^{+} &= w^{+}(p_i + ... + p_n) - w^{+}(p_{i+1} + ... + p_n) \;\; \text{with} \;\; 0 \leq \text{i} \leq \text{n-1} \\ \pi_i^{-} &= w^{-}(p_{-m} + ... + p_i) - w^{-}(p_{-m} + ... + p_{i-1}) \;\; \text{with} - m \leq i \leq 0 \end{aligned}$$

with $w^{+}(0) = 0 = w^{-}(0)$ and $w^{+}(1) = 1 = w^{-}(1)$

Tversky and Kahneman (1992) proposed the following functional form for the value function:

$$v(x) = \left\{ \begin{array}{ll} x^{\alpha} & \text{if } x > 0 \\ -\delta(-x)^{-\beta} & \text{if } \ x < 0 \end{array} \right\} \tag{3}$$

The parameter λ describes the degree of loss aversion (Köbberling and Wakker, 2005). Based on experimental evidences, Tversky and Kahneman estimated the values of the parameters α, β, and δ $\alpha = \beta = 0.88$ and $\delta = 2.25$.

They proposed the following functional form for the weighting function:

$$w^{+}(p) = \frac{p^{\gamma+}}{[p^{\gamma+} + (1-p)^{\gamma+}]^{1/\gamma+}} \qquad w^{-}(p) = \frac{p^{\gamma-}}{[p^{\gamma-} + (1-p)^{\gamma-}]^{1/\gamma-}} \tag{4}$$

Tversky and Kahneman estimated the parameters γ^{+} and γ^{-} as 0.61 and 0.69.

3 Optimal Design in CPT framework

3.1 The optimal structure of payments

3.1.1 Theoretical framework

Our aim is to determine the optimal structure of payments of any lottery-linked-deposit-accounts. We are looking for the structure of payments that maximizes the satisfaction of investors given an expected return (cost) equal to the risk-free interest rate. In this framework, the savings account will be optimal for investors, and thus very attractive to them. And as many investors will subscribe to the contract proposed by the issuer of the asset, the administrative and management costs will be spread over more clients. This will inevitably lead to economies of scale. Consequently, the issuer will have two advantages. On one hand, its expected cost will be the risk-free interest rate; a low interest rate. On the other hand, he will benefit from the economies of scale.

Let's consider a lottery-linked-financial-asset that provides n prizes. Every dollar invested in the asset gives the possibility to subscribers to win one of the n payments.

Let $Y = \begin{pmatrix} y_1 \\ y_2 \\ \\ y_n \end{pmatrix}$ be the payments of the lottery

Let $P = \begin{pmatrix} p_1 \\ p_2 \\ \\ p_n \end{pmatrix}$ be the probability distribution of the lottery[1].

In order to determine the optimal structure of payments, we maximize the satisfaction of investors given an expected cost equal to the risk-free interest rate (r_f). Let $V(X)$ be the subjective utility of the lottery (the satisfaction of investors). $V(X)$ is given by:

$$\begin{aligned} V(X) &= \pi_1 \left[-\delta(r - y_1)^{\alpha}\right] + \sum_{i=2}^{n} \pi_i (y_i - r)^{\alpha} \qquad (5) \\ \text{where } \pi_1 &= w^-(p_1) \\ \pi_i &= w^+(\sum_{j=i}^{n} p_j) - w^+(\sum_{j=i+1}^{n} p_j) \\ \pi_n &= w^+(p_n) \end{aligned}$$

$\pi = \begin{pmatrix} \pi_1 \\ \pi_2 \\ \\ \pi_n \end{pmatrix}$ is the vector of decision weights.

r represents the investors' reference point. We remind that under CPT utility is defined by gains and losses, so a risky prospect is evaluated relatively to a reference point. A payoff less than r is considered as a loss and a payoff greater than r is considered as a gain. In this study, we assume that the first payment, y_1, is the loosing prize of the lottery. It is then

[1]The probability distribution is considered as an exogenous data the issuer cannot modify.

inferior to the reference point r and represents a loss for investors. Therefore the potential loss relatively to the reference point $(r - y_1)$ is weighted by the loss aversion coefficient (δ). We also assume that the $n-1$ other payments $(y_2, ..., y_n)$ are winning prizes. They are thus greater than the reference point. In this study, we class the reference point as the long term interest rate.

The constrained maximization program is given by:

$$\underset{y_i}{Max}\ \pi_1\left[-\delta(r-y_1)^{\beta}\right]+\sum_{i=2}^{n}\pi_i(y_i-r)^{\alpha} \qquad (6)$$

$$\text{subject to} \begin{cases} \sum_{i=1}^{n} p_i \times y_i = r_f \\ r - y_1 \geq 0 \\ y_i - r \geq 0 \quad, i = 2, ..., n \\ y_i \geq 0 \qquad \forall i \end{cases} \qquad (7)$$

Let $\lambda = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \\ \lambda_{2n+1} \end{pmatrix}$ be the Lagrange multipliers..

We can form the Lagrangian:

$$\begin{aligned} L(Y,\lambda) &= \pi_1\left[-\delta(r-y_1)^{\beta}\right]+\sum_{i=2}^{n}\pi_i(y_i-r)^{\alpha}-\lambda_1[y_1-r] \qquad (8) \\ &\quad -\sum_{i=2}^{n}\lambda_i(r-y_i)+\sum_{i=1}^{n}\lambda_{n+i}\times y_i-\lambda_{2n+1}(\sum_{i=1}^{n}p_i\times y_i-r_f) \end{aligned}$$

Thus there exist $\lambda^{*} = (\lambda^{*}_{k}, .k = 1, ...2n+1)$ such as:

$$\left\{\begin{array}{ll}
\dfrac{\partial L}{\partial y_i} = 0 & \forall i = 1, ..., n \\
\lambda_i \times (y_i - r) = 0 & \forall i = 1, ..., n \\
\lambda_{n+i} \times y_i = 0 & \forall i = 1, ..., n \\
\displaystyle\sum_{i=1}^{n} p_i \times y_i = r_f & \\
 & \forall i = 1, ..., 2n+1 \\
\lambda_i \geq 0 & \\
r - y_1 \geq 0 & \\
y_i - r \geq 0 & \forall i = 2, ..., n \\
y_i \geq 0 & \forall i = 1, ..., n
\end{array}\right. \tag{9}$$

with:

$$\frac{\partial L}{\partial y_1} = \pi_1 \times \beta \times \delta (r - y_1)^{\beta - 1} - \lambda_1 + \lambda_{n+1} - p_1 \times \lambda_{2n+1} \tag{10}$$

$$\frac{\partial L}{\partial y_i} = \pi_i \times \alpha (y_i - r)^{\alpha - 1} + \lambda_i + \lambda_{n+i} - p_i \times \lambda_{2n+1} \ , \ \ i = 2, ...n \tag{11}$$

The optimal solution is given by[2]:

[2]Details of resolution are available from the author.

$$
\begin{aligned}
y_1 &= 0 \\
y_i &= r + \left[\left(\frac{p_i}{\alpha \times \pi_i} \right)^{\frac{1}{\alpha - 1}} \times \left(\frac{r_f - (1 - p_1) \times r}{\sum_{j=2}^{n} (\frac{p_j}{\alpha \times \pi_j})^{\frac{1}{\alpha-1}} \times p_j} \right) \right], \quad i = 2, ..., n
\end{aligned} \qquad (12)
$$

3.1.2 Application to Premium Bonds

To comment this result, we can apply it to the Premium Bonds, which are one of the most popular lottery-linked-financial-asset available at this period. Premium Bonds are investments in which instead of getting an interest rate, investors have the chance to win tax-free prizes. For each pound invested, investors receive one bond. Each bond automatically enrolls the holder in a monthly lottery. The prize fund for each draw is shared between three prize bands (higher value, medium value and lower value) and prizes range from £50 to £1 million. Each month's prize fund is calculated relatively to one month's interest determined by the Treasury. For example, in February 2006, the interest rate determined by the Treasury was 0.25% (3% per year). The prizes were allocated in such a way that the expected value of the lottery was 0.25%. Table 1 shows the percentage share of the fund allocated to each prize band, together with the number of prizes, their value and the corresponding probabilities of winning for February 2006.

The application of CPT to Premium Bonds underlines that investors would prefer holding these lottery bonds rather than regular assets that provide a fixed annual interest rate of 4.05%[3] (Pfiffelmann, 2007b). Although these bonds are very attractive to investors, their structure of payments can be modified so that investors' satisfaction is maximal. With the results obtained previously, we can determine for which structure of

[3] The long term interest in 2006

Table 1: Premium Savings Bonds - February 2006

Prize band	**Prize value**	**Number of prizes**	**Probability**
Higher value	£1 million	2	6.7693×10^{-11}
7% of prize fund	£100 000	6	2.0308×10^{-10}
	£50 000	13	4.4001×10^{-10}
	£25 000	26	8.8002×10^{-10}
	£10 000	64	2.1662×10^{-9}
	£5 000	126	4.2642×10^{-9}
Medium value	£1 000	1 772	5.9976×10^{-8}
6% of prize fund	£500	5 316	1.7993×10^{-7}
Lower value	£100	61 530	2.0826×10^{-6}
87% of prize fund	£50	1 162 176	3.9336×10^{-5}
	£0	29 543 512 969	0.999987
Total value	£73.9 million	29 544 744 000	

prizes $Y = \begin{pmatrix} y_1 \\ y_2 \\ \\ y_{11} \end{pmatrix}$ these bonds provide the highest satisfaction to investors.

Table 2 displays the optimal structure of payments of one Premium Bonds.

Table 2: Optimal structure of payments of Premium Bonds under CPT

Probabilities	**Prize value February 2006**	**Optimal Prizes**
6.7693×10^{-11}	£1 million	£36 795 327
2.0308×10^{-10}	£100 000	£41 727
4.4001×10^{-10}	£50 000	£1 127
8.8002×10^{-10}	£25 000	£67
2.1662×10^{-9}	£10 000	£4.42
4.2642×10^{-9}	£5 000	£0.333267
5.9976×10^{-8}	£1 000	£0.004684
1.7993×10^{-7}	£500	£0.003383
2.0826×10^{-6}	£100	£0.003375
3.9336×10^{-5}	£50	£0.003375
0.999958333	£0	£0

The optimal structure of payments is highly skewed. The amount of the jackpot goes from 1 million to more than 35 million and the small and medium prizes have nearly disappeared. The results derived here suggest that investors are willing to accept a decrease of value in the medium prizes in order to increase the value of higher prizes. An explanation of this result lies on the fact that investors are motivated by hope: people dream of growing rich in a sizeable way. In order to have a chance to access higher social standing, they are willing to take risks and give up security. This explains why the issuers of the Premium Bonds should modify the structure of payments by reducing the value of medium prizes in order to increase the value of high prizes. This modification would improve the attractiveness of the bonds. The subjective utility $V(X)$ goes from 0.2671 to 2.84.

The results derived here are consistent with the single account version of the behavioral portfolio theory (BPT-SA) developed by Shefrin and Statman (2000). According to Lopes (1987), three factors have to be taken into consideration in investors' problem of choice: security, potential and aspiration. Aspiration relates to a goal, a target value to reach. Security and potential relate to the principal emotions that operate on investors. On the one hand, investors are driven by fear and wish security for their wealth; they want to avoid poverty. On the other hand, they are willing to take risks in order to become rich. This falls under the concept of potential. Shefrin and Statman developed a portfolio theory by using Lopes' theory of choice under uncertainty. The optimal portfolio in the behavioral portfolio theory is different from the mean variance optimal portfolio. According to Shefrin and Statman, each investor determines a probability of acceptable ruin and a target value that he wants to reach. He then secures his wealth to this target value in almost every state. The number of states secured is determined according to the acceptable probability of ruin each investor has previously established. The remaining wealth is invested in one state. Investors wish to grow rich and bet on one particular state. If this state occurs the portfolio holder will receive much more money than the usual coupon he would have received in the other states. The payoff of this optimal portfolio can be viewed as a combination of bonds, risky or not, and a lottery ticket. The optimal structure of payments we have obtained fits perfectly into this result. Let's consider an investor with an aspiration level equal to the face value of the Premium Bonds and with a probability of ruin equal to zero. In this case, the optimal structure of payments of these bonds is quite similar to the optimal portfolio that would be obtained in behavioral portfolio theory framework. A portfolio composed by Premium Bonds is safe. In every state, bondholders will always get back the face value of the bonds. The investor's principal is secured and their wish of security is therefore satisfied. There are few states where they can win a prize offered by the lottery. Moreover, in one of these states, bondholders can win a jackpot. This chance to win a large amount of money meets everyone's desire to grow rich in a sizeable way. Our results are thus consistent with the conclusions of the behavioral portfolio theory developed by Shefrin and Statman.

Wagenaar (1988), Quiggin (1991) and Shapira and Venezia (1992) also underline the important role of the first prize on the design of lotteries. Both of them indicate that the size of the jackpot has a major effect on the demand of lotteries. However, Shapira and Venezia do not deny the importance of the small and medium prizes on the framework of a lottery. The possibility to win more frequently prevents the subscriber's fatigue from the low likelihood of winning. Clotfelter and Cook (1990) also emphasize the importance of the small and medium prizes. According to them, when a player wins a small prize at the lottery, the satisfaction does not come from the monetary gain but from the possibility to reinvest this small prize in additional lottery tickets. In this way, the player increases their possibility to win the first prize. This possibility to reinvest small prizes can be viewed as playing with house money; and when individuals play with house money they tend to take more risks (Thaler et Johnson, 1990). Thus, in LLDA framework, the gain of a small prize would lead the winner subscriber to reinvest in this asset.

Our results on the optimal design of LLDA underline that investors are willing to accept a decrease of value in the medium and small prizes in order to increase the value of the jackpot. This behavior is therefore consistent with the one observed by Shapira and Venezia. However, the strong decrease of the values of the small and medium prizes runs counter the contributions of Clotfelter and Cook. Actually, the value of the £50, £100, £500 et £1 000 prizes decreases to less than £0.5.

3.2 Optimal payments and optimal probabilities

Let's consider, as previously stated, a lottery-linked-financial-asset that provides n prizes. Each bond provides the possibility for subscribers to win one of the n payments. We assume that the first payment, y_1, is the loosing prize of the lottery and that the $n-1$

other payments (y_2, ..., y_n) are winning prizes. Contrary to previously, we do not consider that the issuer cannot modify the probabilities associated to each payment. Our aim is now to determine for which payments and probabilities the investors satisfaction is the highest. The constrained maximization program is given by:

$$\underset{p_i,\, y_i}{Max} \quad w^{-}(p_1) \times \left[-\delta(r-y_1)^{\beta}\right] + \sum_{i=2}^{n-1} \left[w^{+}(\sum_{j=i}^{n} p_j) - w^{+}(\sum_{j=i}^{n} p_j) \right] \times (y_i - r)^{\alpha} + w^{+}(p_n) \times (y_n - r)^{\alpha} \tag{13}$$

$$\begin{cases} \sum_{i=1}^{n} p_i \times y_i = r_f \\ \sum_{i=1}^{n} p_i = 1 \\ r - y_1 \geq 0 \\ y_i - r \geq 0 \quad , \; i = 2, ..., n \\ y_i \geq 0 \quad \forall i \\ p_i > 0 \quad \forall i \\ p_i < 1 \quad \forall i \end{cases} \tag{14}$$

with

$$w^+(p) = \frac{p^{\gamma+}}{[p^{\gamma+} + (1-p)^{\gamma+}]^{1/\gamma+}}$$

$$w^-(p) = \frac{p^{\gamma-}}{[p^{\gamma-} + (1-p)^{\gamma-}]^{1/\gamma-}}$$

$V(X)$ can then be rewritten as:

$$\begin{aligned} V(X) &= \left[\frac{p_1^{\gamma-}}{[p_1^{\gamma-} + (1-p_1^{\gamma-})]^{1/\gamma-}}\right] \times \left[-\delta(r-y_1)^{\beta}\right] \\ &+ \sum_{i=2}^{n-1}\left[\frac{\sum_{j=i}^{n} p_j^{\gamma+}}{[\sum_{j=i}^{n} p_j^{\gamma+} + (1-\sum_{j=i}^{n} p_j^{\gamma+})]^{1/\gamma+}} - \frac{\sum_{j=i+1}^{n} p_j^{\gamma+}}{[\sum_{j=i+1}^{n} p_j^{\gamma+} + (1-\sum_{j=i+1}^{n} p_j^{\gamma+})]^{1/\gamma+}}\right] \\ &\times (y_i - r)^{\alpha} + \left[\frac{p_n^{\gamma+}}{[p_n^{\gamma+} + (1-p_n^{\gamma+})]^{1/\gamma+}}\right] \times (y_n - r)^{\alpha} \end{aligned} \tag{15}$$

The maximization problem is now more complex: both probabilities and payments must be determined. In order to discuss the optimal design of LLDA we realize some simulations[4]. The subjective utility $V(X)$ seems to increase infinitely when the jackpot (y_n) tends towards infinity with an infinitesimal chance of winning (p_n tends toward zero). In fact, we have:

$$\underset{\substack{p_n \longrightarrow 0 \\ y_n \longrightarrow +\infty}}{Lim} V(X) = +\infty$$

[4]The details of the simulations are given in appendix.

The solution derived here suggests that an infinite jackpot associated to an extremely small probability should be sufficient to attract many investors. Thus, the amount of the jackpot would be the principal determinant of risk-seeking behavior, and the role of the second prizes would be minor. But why have we obtained such a result? The explanation lies on the shape of the weighting function.

Numbers of experiments underline the tendency of individuals to overweight small probabilities. This overweighting of small probabilities induces risk seeking behavior in the domain of gains. The probability weighting function permits probabilities to be weighted nonlinearly. Lots of empirical studies have been done to determine the shape of the weighting function (Lattimore and *al*, 1992; Wu and Gonzales, 1996; Prelec, 1998; Kilka and Weber, 2001). The weighting function is represented, most of time, by an inverse S-shaped (concave in the range $(0, p^*)$ and convex in the range $(p^*, 1)$). In our study we use the specification proposed by Tversky and Kahneman. But, we find the same type of results with the weighting function proposed by Prelec characterized by: $w^+(p) = \exp[-\beta^+(-\ln(p))^\alpha]$. The slope of these specifications tends towards infinity, when the probabilities are extremely small (near 0). Therefore, the lower the probability, the more important the overweighting is. An extremely small probability can thus be infinitely overweighted. As the value function is unbounded, there are situations for which the subjective value of a consequence weighted by its decision weight can be infinitely high. This explains why we have found an infinity jackpot with an infinitesimal chance of winning.

4 Optimal design with a new weighting function

We propose to replace the weighting function specified by Tversky and Kahneman with another functional form whose slope at zero is finite, which still satisfies the overweighting of small probabilities, the underweighting of moderate and high probabilities and the decreasing sensitivity principle. The polynomial form, (Pfiffelmann, 2007a) presented below, avoids subjective values for subjective utility, because its slope at zero is finite, and satisfies the conditions describe above.

$$
\begin{aligned}
w(p) &= 2215.003p - 19080.29p^{1.1} + 30702.47p^{1.15} - 13963.8p^{1.2} \\
&\quad +202.1941p^{2} - 76.95053p^{2.5} + 2.37243p^{6}
\end{aligned} \tag{16}
$$

If we realize the same simulations as the ones realized previously, the satisfaction of investors does not increase infinitely with the value of the first prize[5]. From a threshold, investors are not willing to reduce the values of the small and medium prizes in order to increase the jackpot of the lottery. An infinite first prize associated to an extremely small probability is not sufficient to attract a lot of investors. The replacement of the weighting function then permits to obtain a more realistic result.

It would be interesting to determine the optimal structure of payments of lottery-linked-financial-assets with this new weighting function. As the probabilities are differently transformed (near 0 and 1) with this polynomial weighting function, the optimal structure of payments should be different. Table 3 represents the optimal structure of payments of one Premium Bond. The first column displays the original structure of payments provided by the issuer in February 2006. The second column displays the results we have obtained in the previous section (the optimal structure of payments with the weighting function specified by Tversky and Kahneman). The third column represents the optimal structure of payments when the weighting function is the polynomial functional presented above. The new optimal structure of payments is less skewed and dispersed than the one obtained previously. The first prize is now equal to £2.8 million instead of the £36 million obtained previously. This result lies on the fact that the overweighting of small probabilities, the overweighting of the event "winning the first prize," is less important with the new weighting function. The decision weight associated to this event is now about 7.9×10^{-8} (for a probability equal to 6.7693×10^{-11}), while it was equal to 6.26087×10^{-7} with the original weighting function (so 8 times less). Therefore, if we take into account a weighting function whose slope is finite at zero, the optimal structure of payments will not devote almost all the prize fund to the jackpot. With this new structure of payments the medium prizes

[5] The details of the simulation are given in appendix.

Table 3: Premium Bonds

	Original Payments	**Optimal Payments** w **CPT**	**Optimal Payments** w **polynomiale**
y_1	0	0	0
y_2	50	0.003375	0.11
y_3	100	0.003375	65
y_4	500	0.003383	1 630
y_5	1 000	0.004684	10 766
y_6	5 000	0.333267	65 728
y_7	10 000	4.42	133 085
y_8	25 000	67	262 360
y_9	50 000	1 127	494 576
y_{10}	100 000	41 727	1 020 835
y_{11}	1 000 000	36 795 327	2 882 626

have not disappeared and are now relatively significant. However, the first prize is still higher than the one really provided by the issuer. This optimal structure of payments still underlines he major role of the first prize on the design of LLDA.

These results are consistent with the multiple accounts version of the behavioral portfolio theory (Shefrin and Statman, 2000). In the BPT-MA version, Shefrin and Statman integrate the mental accounting structure from Kahneman and Tversky prospect theory. They take into account that most investors combine low aspiration and high aspiration levels. Thus, they act as if they create mental accounts and apply specific decision rules to each account, segregating their portfolio into distinct mental accounts (Thaler, 1985). For each account, investors secure their wealth to the target value corresponding to this account. They then invest the remaining wealth in one particular state. The optimal portfolio of each account consists mainly on bonds (risky or not) and lottery tickets, but "the accounts are pushed in the layers to the extremes" (Shefrin and Statman, 2000). The account that corresponds to the lowest aspiration level is mainly made up with risk-free bonds, whereas the highest aspiration level account is more like a lottery ticket. The optimal portfolio can be view as a combination of bonds that intend to satisfy several aspiration levels and lottery tickets. The optimal structure of payments we have obtained for LLDA perfectly fits into this result. Each payment corresponds to one aspiration level where the lowest one can be assimilated to the status quo.

5 Discussion

In this study, we have investigated the optimal design of lottery-linked-deposit-accounts given that all investors have Tversky and Kahneman's individual preferences. The popularity of these financial assets cannot be understood in light of traditional models of rational choices such as expected utility theory. It has been observed that investors transform objective probabilities via a weighting function, which overweights the tails of a probability distribution. This observation cannot be integrated in the framework of expected utility theory. Under cumulative prospect theory, people express risk seeking behavior for low probability gains. We show that such a behavior can explain the development of these financial accounts. In this study, we determine the optimal structure of payments of these financial assets under CPT framework. The analysis presented here suggests that the lottery should be strongly asymmetrical. In order to attract a maximum number of investors, banks should provide financial assets that are very positively skewed. The optimal structure of payments derived in this study is then consistent with the behavioral portfolio theory developed by Shefrin and Statman (2000). On the one hand, LLDA are "riskless": the lottery does not affect the principal but only the interest rate provided by the issuer. Thus, the security desire of investors (driven by fear) is fulfilled. On the other hand, investors can win a large amount of money. The possibility of winning a large jackpot makes investors (driven this time by potential) dream of and meets their expectations of wealth. We also point out that, under CPT, the satisfaction of investors seems to increase infinitely when the jackpot tends towards infinity with an infinitesimally chance of winning. An infinite jackpot associated to an extremely small probability should be sufficient to attract a lot of investors. We suggest that an explanation of the results could lie on the shape of the weighting function. In our study, we use an inverse S-shape weighting function (the one formed by Tversky and Kahneman) because this function provides a good explanation of risk taking behavior. However, the slope of this function tends towards infinity when the probability of winning is extremely small. Thus, obtaining such results is not amazing, but is it realistic to imagine a financial asset that provides only an infinite gain with an infinitesimally chance of winning?

We then propose to replace the weighting function specified by Tversky and Kahneman with a polynomial functional that preserves the main characteristics of the inverse S-shape specification but whose slope at zero is finite. Resorting to this functional permits to avoid infinite subjective utility for any LLDA. The satisfaction of investors does not anymore increase infinitely with the value of the first prize. The optimal structure of payments we have obtained in this new framework still suggests that the lottery should be strongly asymmetrical to attract a maximum number of investors, but it does not challenge to role of the medium prizes in the design of lotteries.

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6 Appendix

Table 4: Simulations -Tversky and Kahneman weighting function

y_1	y_2	y_3	y_4	p_1	p_2	p_3	p_4	**V(X)**
0	11.9	26.8	479	0.9998	5.79×10^{-5}	5.79×10^{-5}	5.79×10^{-5}	**0.5**
0	206	213	2345	0.99996	1.08×10^{-5}	1.08×10^{-5}	1.08×10^{-5}	**0.8**
0	900	10 000	2 800 000	0.9999989	1×10^{-6}	1×10^{-7}	1×10^{-8}	**6.3**
0	900	10 000	28 000 000	0.9999989	1×10^{-6}	1×10^{-7}	1×10^{-10}	**11.6**
0	5	6	29 900 000	0.9999989	1×10^{-6}	1×10^{-7}	1×10^{-10}	**12.2**
0	5	6	2.99×10^{11}	0.9999989	1×10^{-6}	1×10^{-7}	1×10^{-13}	**147.3**
0	4	4	2.99005×10^{11}	0.9999989	1×10^{-6}	1×10^{-7}	1×10^{-13}	**147.4**

Table 5: Simulations - Polynomial weighting function

	y_1	y_2	y_3	y_4	p_1	p_2	p_3	p_4	**V(X)**
s_1	0	5	10	299 940	0.9999988	1×10^{-6}	1×10^{-7}	1×10^{-7}	**3.76**
s_2	0	5	10	2 999 400	0.99999889	1×10^{-6}	1×10^{-7}	1×10^{-8}	**3.77**
s_3	0	5	10	29 999 400	0.999998899	1×10^{-6}	1×10^{-7}	1×10^{-9}	**3.53**
s_4	0	5	500	2 993 599	0.99999887	1×10^{-6}	1×10^{-7}	1×10^{-8}	**3.78**
s_5	0	5	500	2 993 599	0.999999989	0	0	1.02×10^{-8}	**3.76**

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