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Does the consciousness of the disposition effect increase the equity premium?

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The disposition effect is a well established phenomenon in the empirical and experimental financial literature. It leads to sell winners too early and to hold losers too long. In this paper, we show that the consciousness of the disposition effect by investors lead them to require a greater risk premium to invest in stocks (when compared to rational investors). We also analyze the role of the evaluation period for disposition investors. We show that the risk premium they require is a decreasing function of the delay between two evaluations of their portfolio. The influence of the evaluation period on the equity premium looks like the one induced by myopic loss aversion (Benartzi-Thaler, 1995) but the origin is different. Valuing more often a portfolio give more occasions to sell winning stocks and then decreases the expected return. This point is analyzed by assuming that returns are driven by a Brownian motion and that investors evaluate their portfolio at regularly spaced dates.

Classification JEL: G11, G14
Key words: disposition effect; equity premium puzzle; loss aversion; behavioral finance

1. Introduction

The disposition effect is the tendency of investors to sell winning stocks too early and to hold losing stocks too long. It was first analyzed by Shefrin and Statman (1985) and confirmed on individual data by Odean (1998), among others. Experimental evidence of the disposition effect has also been obtained by Weber and Camerer (1998).

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Selling winning stocks too early can refer to self-control problems, to an irrational belief in mean reversion of prices, or to the fact that investors seek pride and want to avoid regret (Muermann and Volkman, 2006). It also suggests a possible time-inconsistency in successive decisions. It is as if investors were changing their horizon of investment, depending on the evolution of stock prices. Such investors are usually called *disposition investors*. The empirical evidence shows that disposition investors dynamically revise their portfolios in a sub-optimal way. Thus, even if an investor decides to buy stocks at date 0 for several periods, she may be conscious that the final probability distribution of her wealth will not be the final distribution of the value of the portfolio set up at date 0. This leads to the question we address in this paper. Does the consciousness of the disposition effect generate an increase in the equity premium required by an investor? And if it is the case, is the risk premium variation sizeable?

To the best of our knowledge, this question has never been directly addressed in the literature. The influence of the disposition effect on the equity premium has been analyzed through the role it plays in changing the demand function of disposition investors. In particular, Grinblatt and Han (2005) present a model in which current prices are jointly determined by the fundamental value of the stock and by the reference price of the investor (the initial buying price in the standard case). Working on the database of Odean (1998), Goetzmann and Massa (2003)[2] test the model of Grinblatt and Han and show that the existence of disposition investors has a non negligible effect on prices and returns.

In this paper, we first consider a simple two-period model of an exchange economy where two assets are traded: a risky asset, called the stock, and a risk-free asset, called the bond. The price of the risky asset is assumed to be driven by the usual Cox-Ross-Rubinstein model (1979). Agents' preferences are described by a simplified version of prospect theory[3]. Investors are characterized by a piecewise linear valuation function so that the expectation of their value function can be expressed as the terminal value of a portfolio of call and put options.

[2] The paper by Grinblatt and Han was already circulating as a NBER working paper in 2002.
[3] Kahneman and Tversky (1979) and Tversky and Kahneman (1992).

We introduce the disposition effect by assuming a strictly positive probability of selling the stock at date 1 when an up-state occurs. To simplify the formulation of the model, we assume that the probability of switching from stocks to bonds after a down-state is equal to 0. We could obtain similar results by defining a probability of switching in each state as long as the switching probability in the up-state is greater than the corresponding one in the down-state.

We then extend the model to a multi-period framework by assuming a geometric Brownian motion for the stock price and regularly spaced valuation dates. We estimate the parameters of the probability distribution of realized returns by Monte Carlo simulations.

Our main results are the following:

1. Disposition investors reach a lower level of welfare and the loss of welfare increases with the intensity of the disposition effect and the frequency at which the portfolio is evaluated.
2. When investors find optimal to invest in stocks for one period, they also find optimal to invest in stocks for two periods, even if they are conscious of being disposition investors.
3. Conscious disposition investors require a higher risk premium and the increase in this risk premium is negatively linked to their evaluation period

The paper is organized as follows. In section 2, we present the related literature on the disposition effect. We recall the essential empirical and experimental results showing the presence of this effect among individual and professional investors.

In section 3, we first analyze the one-period problem. We define what we label a *critical investor*, that is an agent characterized by a (critical) loss aversion parameter which keeps her indifferent between stocks and bonds. We then study, in a two-period model, the relationship between this critical loss aversion parameter and the horizon of investment and show that it is decreasing. This result is an argument in favour of time-diversification, even if the reference wealth level is chosen as the initial wealth capitalized at the risk-free rate, as in Barberis *et al.*(2001). The conclusion is obviously reinforced if a lower reference wealth level is chosen.

In section 4, we introduce the disposition effect in the two-period model. We assume that there is a positive probability of switching from stocks to bonds at date 1, after a stock price increase in the first-period. We show that the risk premium required by investors is an increasing function of the intensity of the disposition effect.

In section 5, we propose the corresponding multi-period model and simulate the behaviour of disposition investors. We confirm that the expected return they obtain is a decreasing function of the parameter describing the intensity of the effect. However, the variance of returns is also a decreasing function of the same parameter. An analysis of the Sharpe ratio and of the level of the valuation function is then realized to draw some conclusions in terms of performance.

Section 6 concludes the papers. All the proofs are reported in the appendix.

2. Related literature

2.1 What drives the disposition effect?

Though our essential purpose is not to analyze the theoretical explanations of the disposition effect, it is worth to stress some of the arguments usually referred to in the literature to explain this phenomenon. The question in the title of this subsection appears in the title of two papers by Zuchel (2001) and Barberis and Xiong (2006). In fact, following Shefrin and Statman (1985), a number of authors (for example Odean (1998) and Weber and Camerer (1998)) have justified the disposition effect by prospect theory (PT) preferences, especially by the S-shape valuation function assumed in PT. The argument is, roughly speaking, the following: after a gain (loss), agents are in the concave (convex) part of the valuation function so they become risk averse (lovers).

When agents are risk-averse over gains and risk lovers in the domain of losses, they prefer to realize paper gains and keep paper losses. However, the above arguments are quite informal; Barberis and Xiong (2006) (see also Hens and Vlcek (2005)) have analyzed the theoretical foundations of this explanation. They get controversial results. They show that the disposition effect is observed for some values of the essential variables (the expected stock return and the horizon), but also find the opposite effect for other reasonable values of these variables. The intuition of their result, also found in Hens and Vlcek, may be simply summarized in a two-period framework. Investors initially buying stocks for two periods and prone to sell their stocks after a gain at date 1, would not have bought the stock in the first place at date 0. These authors exhibit, in a number of situations, a contradiction between the first purchase at date 0 and the decision to sell at date 1. In a continuous-time model, Kyle *et al.*(2006) also show that, depending on the risk-return characteristics of the stocks, the disposition effect or the opposite can arise at the theoretical level. Roughly speaking, investors exhibit a propensity to hold on to losers for stocks with relatively low Sharpe ratios. They may liquidate early their winning stocks when the current profits are rising or when they drop to the break-even point.

An other possible explanation of the disposition effect is found in the psychological literature on *entrapment* or *escalation of commitment*. This literature tries to understand why and under what conditions people stick or reinforce failing courses of action[4]. Entrapment appears in dynamic settings when agents face negative feedbacks about past decisions and have to choose between stopping or pursuing a course of action. In an investment context, the question is to know if it is better to stick with a losing investment, to increase the stake or to sell the losing securities and choose other ones to invest in. For practitioners, increasing the stake is sometimes interpreted as using a *dollar cost averaging* strategy, that is to say, buying more stocks after a price decrease to lower the mean buying price. It has been shown in different contexts that this strategy is dominated by the usual buy and hold strategy, but it remains very popular.

Psychologists explain entrapment by the need to justify prior decisions; it is called the *self justification hypothesis* (Brockner, 1992). When an investor buys a given stock and later observes paper losses, she may be reluctant to realize these losses because it is equivalent to admit that the first decision was a mistake. This interpretation may also be linked to cognitive dissonance theory (Festinger, 1957). Unlucky investors rationalize ex post their initial decision by using arguments like “the price will bounce back”. In other words, they start to irrationally believe in mean reversion (see Shu *et al.*(2005) for such an interpretation).

The results of these diverse approaches show that there is no consensus about the explanation of the disposition effect. However, as we will see in the next section, this is a very robust phenomenon on the empirical and experimental points of view.

2.2 The empirical evidence

The label *disposition effect* was first given by Shefrin and Statman (1985) to translate the tendency of investors to sell the winners too early and to hold on to losers too long. The early studies analyzed the abnormal trading volumes on stocks that had risen in price over previous periods. Lakoniskok and Smidt (1986) found much more volume for winners on NYSE and Amex stocks and Ferris *et al.* (1988) showed on 30 U.S stocks that current volume is negatively correlated with volume on the preceding days, when the price has risen. The reverse correlation appears for stocks whose prices were lower on the preceding days. As these studies were concerning aggregate volume, they were not revealing the individual decision process of buyers and sellers.

Concerning the behaviour of individual investors, the reference study is Odean (1998) who analyzed 10 000 individual accounts at a large discount broker between 1987 and 1993. The two main results of Odean are the following:

a. The proportion of realized gains is significantly larger than the proportion of realized losses, except in December (essentially for tax reasons).
b. The winning stocks (which were sold) perform better, in subsequent periods, than the losing stocks (which were kept).

An elementary interpretation of the first result is that a paper loss is perceived as less painful than a realized loss and/or that a realized gain generates more utility than a paper gain. The other usual interpretations are that investors erroneously believe that stock prices revert to the mean; they then realize

[4] The decision by President Bush to send 20 000 more soldiers in Iraq at the beginning of 2007 is an illustration of this kind of decision process. Akerlof (1991, p15) gives a similar example about the way the President Johnson's

gains and retain losing investments to wait for positive returns, expected to compensate past paper losses. Selling the winners may also be justified to rebalance portfolios or to avoid high transactions costs on low-price stocks. Odean (1998) found that the disposition effect was persistent even when controlling for these two arguments. Lehenkari and Pertunnen (2004) analyzed individual trades on the Finnish market and also found that capital losses reduce the selling propensity of investors; however, they didn’t find the opposite effect for capital gains.

Concerning professional traders, Garvey and Murphy (2004) analyzed the behaviour of a proprietary stock-trading team (15 traders) specialized on NASDAQ stocks. This team generated $1.4 million in intraday trading profits for a 3-month period (8 march-13 june 2000), in a downward-trending market. The members of such teams obviously work with a very short horizon (a few minutes) and their trades are not motivated by diversification needs or capital constraints. The activity of each trader in the team usually focuses on one or two stocks. Garvey and Murphy showed that the mean duration of a losing roundtrip is 268 seconds and the duration of a winning roundtrip is only 166 seconds. When a day was divided into three periods of about two hours each, the difference between the durations of loosing and winning roundtrips remained significant in each period, even if mean durations were much shorter in the morning. When the authors distinguished three categories of roundtrips, depending on the size of the trade, they also found a significant difference between the mean durations in each category. Finally, Garvey and Murphy showed that closing profitable positions early and holding losing positions longer was not optimal because it was reducing the global profitability of the team. Coval and Shumway (2005), Frino and *al.* (2005) and Locke and Mann (2003) obtain the same kind of results on different futures markets. In the same vein, Jordan and Diltz (2004) show that a large majority of day traders hold losing trades longer than profitable trades. Shapira and Venezia (2001) also observe the disposition effect for professional and individual investors on the Israeli market. Shu *et al.* (2005) and Barber *et al.* (2006) show that tawainese investors are much more reluctant to realize their losses than U.S investors. Nevertheless, Shu *et al.*(2005) observe the reverse effect on low-return, high-price stocks. They interpret their findings by saying that

Tuesday lunch group was working during the Vietnam war.

Taiwanese individual investors exhibit a stronger belief in mean reversion than U.S investors. Finally, Genesove and Mayer (2001) illustrate the differences in the behaviour of buyers and sellers on the housing market, leading to conclude to the existence of a disposition effect.

2.3 The experimental evidence

Weber and Camerer (1998) (see also Chui (2001)) conducted an experiment with six stocks characterized by different (but constant) expected returns. The price changes were exogenous so that players could infer (after a number of periods) with reasonable accuracy the stocks with an upward (downward) trend or no trend at all. This design was chosen to mitigate the disposition effect or, in other words, to reinforce the result if this effect was observed. On the aggregate, 51 % of sales were concerning winning stocks and only 39 %, losing stocks.

Two other interesting points have to be mentioned in this study. Participants were informed about the independence of successive price changes. They then could not rationally believe in mean reversion. As they were learning about the trend after each price change, one could have expected that players were eager to keep winning stocks and to sell losing stocks, a behaviour contradicting the disposition effect.
A second experiment presented by Weber and Camerer (1998) was also realized. In this one, stocks were automatically sold at the end of each of the 14 periods of the game. Participants had the possibility to buy back the stocks immediately, without bearing transaction costs. The authors observed that the disposition effect was largely reduced when this automatic selling procedure was used. Players were not so eager to buy back the losing stocks. This observation leads to evoke the question of self-control problems. When agents have to decide themselves, they irrationally believe that stocks will bounce back after a price decrease, even if they were convinced at preceding dates that successive price changes are independent random variables. Roughly speaking, they are subject to the gambler's fallacy. After losing periods, they start to believe that "luck" is due in the next draws.

3 Loss aversion in a discrete-time model

3.1 Preliminaries

We first consider a one-period model with two dates, denoted as 0 and 1. Loss averse investors are characterized by valuation functions defined as follows:

$$v(x) = (x - x^*)_+ - \lambda(x^* - x)_+$$

where x is the final wealth, $\lambda > 1$ is the loss aversion coefficient and x^* is the reference wealth level.

Assume that two assets are traded on the market at date 0: first, there is a stock, the random terminal payment of which being denoted as S, taking N terminal values $(x_i, i = 1,...,n)$ with probabilities $(p_i, i = 1,...,n)$. Second, there is a risk-free asset, the bond, generating a known gross rate of return, denoted as r. The initial price of the bond is normalized to 1 and the terminal values of the risky asset are assumed to be ranked in the increasing order. This assumption is not crucial since the terminal values of any portfolio containing stocks and bonds will be ranked in the same order.

An investor endowed with an initial wealth W_0 chooses to invest in stocks if her expected valuation function is greater to the one obtained by investing in bonds. We assume that the date-1 reference level is rW_0, the terminal wealth level for a 100 % investment in the bond. A consequence of this assumption is that the valuation function is equal to 0 when W_0 is fully invested in bonds.

It allows us to assume, without loss of generality, that the initial wealth is equal to 0. In this framework, an investor who wants to buy stocks, borrows at date 0 at the risk-free rate r – 1. The net cash-flow obtained at the terminal date is then equal to $W_1 - rW_0$ which can be simply written as $N(S_1 - rS_0)$ where S_t, t = 0,1, stands for the date-t price of the stock and N is the number of stocks bought at date t = 0. We assume N = 1 in the following.

As decisions are taken at date 0, the investor bases her evaluation on the probability distribution of her final wealth. Purchasing the stock is then desirable if the following inequality is satisfied:

$$E\left[v(S_1)\right]=\sum_{i=1}^{n}p_i\left(x_i-rS_0\right)_+-\lambda\sum_{i=1}^{n}p_i\left(rS_0-x_i\right)_+\geq 0 \tag{1}$$

When $E\left[v(S_1)\right]=0$, the investor is indifferent between stocks and bonds. Thanks to the ranking of the terminal values of the stock, equation (1) can be written as follows:

$$E\left[v(S_1)\right]=\sum_{i=i_0}^{n}p_i\left(x_i-rS_0\right)-\lambda\sum_{i=1}^{i_0-1}p_i\left(rS_0-x_i\right)\geq 0$$

where the i_0 is defined by $i_0=\inf\left\{i\in\{1,2,...n\}/\,x_i\geq rS_0\right\}$.

The first question we address in the following is the determination of the loss aversion parameter which keeps the investor indifferent between stocks and bonds.

3.2 The critical loss aversion index in the two-state, one-period model

We now assume that $n = 2$, $p_2 = p$ and $p_1 = 1-p$. The first state is called the down-state and the second the up-state. We simply get the following preliminary result.

Proposition 1

1) The loss aversion parameter λ^*, keeping the investor indifferent between the two assets, is defined by:

$$\lambda^*=\frac{p}{1-p}\frac{u-r}{r-d} \tag{2}$$

where u and d are defined by $x_2=uS_0$ and $x_1=dS_0$.

2) $\lambda^*>1\Leftrightarrow E\left(\frac{S_1}{S_0}\right)>r$

u and *d* define the gross returns on the risky asset in the two states. These notations are the ones used by Cox-Ross-Rubinstein (1979) in their seminal paper on the binomial option pricing model.

An agent characterized by the parameter λ^* will be called a *critical investor*.

The formulation of λ^* in equation (2) has a natural interpretation. Assume that the probabilities of the two states are equal to 0.5 and consider a distortion of parameters u and d such that the expected return on the stock remains constant but the variance increases. For example, if u becomes $u + a$ and d becomes $d - a$, the parameter λ^* decreases when a is positive. It simply means that some investors preferring stocks in the first economy would choose bonds in the other, more risky, economy.

The second point in proposition 1 is also intuitive. The existence of a positive risk premium characterizes a loss averse critical investor ($\lambda^* > 1$). In fact, if the critical investor was risk-neutral (or loss-neutral), there would be no reason to get a risk premium on the stock.

Finally, the critical value obtained in proposition 1 has a nice interpretation in terms of probabilities. It can be seen as the ratio of two probability ratios by writing:

$$\lambda^* = \frac{p/(1-p)}{(r-d)/(u-r)}$$

The numerator is the ratio of the real probabilities of the two states and the denominator is the ratio of the corresponding risk-neutral probabilities.

Finally, λ^* is a decreasing function of the risk-free rate. In fact, we have :

$$\frac{\partial \lambda^*}{\partial r} = \frac{p}{1-p}\frac{d-u}{(r-d)^2} < 0$$

All other things being equal, a lower proportion of investors choose the risky asset when the risk premium is lower or, equivalently, when the risk-free asset is more attractive.

Suppose now that the critical investor wants to hedge her portfolio of stocks by buying a put option with an exercise price equal to $K = rS_0$. This exercise price is the one which prevents the investor's wealth to become negative at date 1. The following corollary shows that the reservation price of this put option is equal to the arbitrage-free price obtained in the usual binomial model.

Corollary 2

The reservation price of a put option with a strike price $K = rS_0$ which prevents the critical investor from getting a negative final wealth is equal to:

$$\phi_0 = \frac{1}{r}(r-d)S_0\left(\frac{u-r}{u-d}\right) \tag{4}$$

This is in fact not surprising because the critical investor is indifferent between stocks and bonds. It simply confirms that the critical loss aversion index establishes the link between real and risk-neutral probabilities, as mentioned before. Indeed, the formulation of the put price does not depend on the real probability of a down-state. It is so because, for the critical investor under consideration, the risk of the stock is exactly compensated by its expected return.

3.3 The two-period model

To study the time-diversification effect in the discrete-time model, we consider a two-period binomial model with constant parameters *u* and *d* and *p*. The expected valuation function is:

$$E\left[v(S_2)\right] = p^2(u^2-r^2)+2p(1-p)(ud-r^2)-\lambda^*(1-p)^2(r^2-d^2) \text{ if } ud > r^2$$

$$E\left[v(S_2)\right] = p^2(u^2-r^2)-\lambda^*\left(2p(1-p)(r^2-ud)+(1-p)^2(r^2-d^2)\right) \text{ if } ud \le r^2$$

The following proposition shows that the critical investor of the preceding section would obtain a strictly positive expected valuation function by investing in stocks.

Proposition 3

If $\lambda = \lambda^* > 1$, *we get* $E\left[v(S_2)\right] > 0$.

This result can be interpreted in several ways. First, if we assume that investors are characterized by the same loss aversion parameter, those with a short horizon sell their stocks to investors with longer horizons. However, one cannot be satisfied by this idea since it leads to a quite surprising result. In equilibrium, the only investors possessing stocks are the ones with the longest horizon.

In the preceding two-period model, investors do not decide anything at date 1. However, the market is open and they could change their mind by this date, switching from bonds (stocks) to stocks (bonds). In the following section we introduce the disposition effect by allowing our critical investor to sell her stocks at date 1.

4 The disposition effect and the equity premium

4.1 Introduction of the disposition effect

When investors are prone to the disposition effect, it may have some consequences on the equity premium they require on stocks. In fact, assume that an agent with a given horizon T decides at date 0 to invest in stocks. After a gain (a stock price increase), she may be prone to switch to bonds, even if she decided at date 0 with a two-period horizon in mind. This tendency to sell winners too early depends on several factors like willpower or self-control. However, an investor knowing that she has self-control problems[5] also knows that the distribution of returns she will face at date T is not the one corresponding to a 100 % stock portfolio.

In the following, we introduce this effect in the two-period model of section 3. More precisely, we consider that the investor will switch to bonds with a probability θ at date 1 when the stock price is uS_0.

Figure 1 around here

Figure 1 depicts the two-period tree in this framework. At dates 0 and 1, the lower cell in each group contains the vector of probabilities of reaching the successors, starting by the probability of an up-state at the next date. Compared to the model in section 3, the essential modification is the introduction of the value urS_0 at date $t = 2$, which is possibly reached with a probability θ when starting in the up-state at $t = 1$.

This change in the two-period tree takes into account a switch from stocks to bonds at date 1, due to the disposition effect.

[5] Obviously, the problems we refer to here are not pathological. Everybody has experienced such problems. You take a decision today, quit smoking tomorrow for example, and you continue to smoke the day after, and so on.

Let us assume $S_0 = 1$ to simplify the notations. The value of the portfolio of the investor can now take four values at date 2, respectively u^2, ur, ud and d^2 with probabilities $p^2(1-\theta)$, $p\theta$, $p(1-p)(2-\theta)$ and $(1-p)^2$.

4.2 The disposition effect and the agent's welfare

We now change our notations and denote W_2 as the final wealth of the investor at date 2. This change of notations is justified because an investor starting with stocks may end with bonds at date 2 by switching at date 1 in the up-state. The date-2 stock price is then not her final wealth in this case. We also use $v(W_2, \lambda, \theta)$ to denote the value function because it now depends on the probability of a switch at date 1.

Proposition 4

If the equity premium is positive, we have:

$$\frac{\partial E\left[v(W_2,\lambda^*,\theta)\right]}{\partial\theta} < 0 \ \ and \ \ E\left[v(W_2,\lambda^*,1)\right] > 0$$

In the binomial model, we need to distinguish the two cases $ud > r^2$ and $ud < r^2$. We will first consider $ud > r^2$ which is the most plausible case when the volatility of stock returns is not too high and the expected return not too low[6]. Proposition 4 is intuitive when $ud - r^2 > 0$. In this situation, the date-1 choice is equivalent to play a lottery which generates only gains. As our value function is piecewise linear, it is as if the investor was risk-neutral on this part of the curve. Consequently, the investment in stocks generates value because the equity premium is positive and the investor does not require any risk premium here. It explains why it is in fact not optimal to realize the gain at date 1. When $ud - r^2 < 0$, two

[6] In fact, let $u = r + \pi + \sigma$ and $d = r + \pi - \sigma$ where π stands for the equity premium and assume here that $p = 0.5$. σ is then the standard deviation of returns. We get:

$$ud - r^2 = (r + \pi + \sigma)(r + \pi - \sigma) - r^2 = 2\pi r + \pi^2 - \sigma^2$$

If π is comparable to the historical equity premium (around 6 %) and if we assume that $r = 1$, corresponding to a risk-free rate equal to 0, the volatility of the stock return must be higher than 40 % for the right hand-side of the above equation to be negative. Obviously, this remark is reinforced if the risk-free rate is positive.

conflicting effects are at work. The first one is the equity premium, which is an incentive to hold stocks in the second period. The second one comes from the fact that a price decrease in period 2 generates a loss. It may then be an incentive to switch to bonds because even if a down-state occurs after switching, the final wealth corresponds to a gain equal to $ur - r^2$. Proposition 4 means that the first effect is always greater for the one-period critical investor. At a first glance, it seems surprising; however the result doesn't concern any investor but only the critical investor (and investors with a lower loss aversion coefficient).

The second inequality in proposition 4 shows that the expected valuation function of the one-period critical investor remains positive when she is prone to the disposition effect. In other words, the equity premium puzzle cannot be explained only by the disposition effect in a two-period model.

However, the first part of this proposition implies that the equity premium required by disposition investors is larger when they take into account the disposition effect. The following subsection addresses the question of the measurement of the variation in the equity premium, coming from the existence of this effect.

4.3 The variation of the equity premium due to the disposition effect

The purpose of this section is to compare the risk premia required by two investors characterized by the same loss aversion coefficient. More precisely, we will take as a benchmark an investor who is not prone to the disposition effect and determine the risk premium required by a second investor prone to this effect with a probability θ.

Our benchmark investor is the **two-period** critical investor characterized by a loss aversion parameter $\lambda = \lambda^{'} > 1$. To allow a direct comparison of risk premia, we consider the simple case $p = 0.5$, $u = r + \pi + \sigma$ and $d = r + \pi - \sigma$. The parameter π is then the risk premium and σ is the standard deviation of stock returns.

Proposition 5

The critical two-period loss aversion parameter $\lambda^{'}$ is defined by:

$$\lambda' = \frac{4\pi^2 - (\sigma-\pi)^2 + 2r(3\pi+\sigma)}{(\sigma-\pi)(2r+\pi-\sigma)} \text{ if } ud - r^2 > 0 \qquad (5)$$

$$\lambda' = \frac{4\sigma(r+\pi)}{(\sigma+\pi)^2 - 4\pi^2 + 2r(\sigma-3\pi)} \text{ if } ud - r^2 < 0 \qquad (6)$$

In footnote 6 we can observe that the relationship $ud = r^2$ induces the following link between the volatility and the risk premium:

$$\sigma^2 = \pi(2r+\pi) \qquad (7)$$

In this special case, the value of $\lambda^{'}$ is given by:

$$\lambda' = \frac{(\sigma+\pi)(2r+\pi+\sigma)}{(\sigma-\pi)(2r+\pi-\sigma)} \qquad (8)$$

To estimate the critical loss aversion parameter, consider the following figures reported in Barberis *et al.* (2001) concerning Treasury Bills and NYSE stock data on the period 1926-1995:

$$r = 1.0058; \pi = 5.45\%; \sigma = 20.02\%$$

These values correspond to the first part of proposition 5, that is $ud - r^2 > 0$. We obtain $\lambda^{'} = 2.65$, a value very close to the estimation by Benartzi and Thaler (1995) for a piecewise linear utility function (2.77), but slightly greater than the one estimated by Tversky and Kahneman (1992) which was 2.25 (but obtained with an S-shape valuation function).

4.4 Sensitivity analysis

We now analyze the supplementary premium required by an investor characterized by $\lambda^{'} = 2.65$, if she is prone to the disposition effect with intensity θ. In other words, we study the relationship between θ and π,the other parameters being given. According to the historical levels of the risk-free rate and to the volatility of stock returns, we only consider the case $ud - r^2 > 0$.

The expected valuation function may now be written as:

$$
\begin{aligned}
E\left[v(W_2,\lambda',\theta)\right] &= 0.25((1-\theta)\left[(r+\pi+\sigma)^2-r^2\right]+2\theta r(\pi+\sigma) \\
&+(2-\theta)\left[(r+\pi+\sigma)(r+\pi-\sigma)-r^2\right]+\lambda'\left[(r+\pi-\sigma)^2-r^2)\right]) \\
&= 0.25((1-\theta)\left[(\pi+\sigma)^2+2r(\pi+\sigma)\right] \\
&+2\theta r(\pi+\sigma)+(2-\theta)(\pi^2-\sigma^2+2r\pi) \\
&-\lambda'\left[(\pi-\sigma)^2+2r(\pi-\sigma)\right]) \\
&= 0.25((1-\theta)(\pi+\sigma)^2+2r(\pi+\sigma) \\
&+(2-\theta)(\pi^2-\sigma^2+2r\pi)+\lambda'\left[(\pi-\sigma)^2+2r(\pi-\sigma)\right])
\end{aligned}
$$

(9)

The numerical results are summarized on figure 2 which represents the equity premium required by the critical investor as a function of θ. Using the abovementioned parameters, we get a risk premium varying from 5.45 %, when $\theta = 0$, to 7 % when $\theta = 1$, that is a premium variation of 1.5 %. However, the intensity of the disposition effect cannot be expected to be very strong, corresponding to values of θ around 0.2 if we refer, for example, to the experiments of Weber and Camerer (1998). It leads to an increase of about 25 basis points for the risk premium. Consequently, even if the role of the disposition effect is not negligible, the disposition effect cannot justify a significant part of the equity premium in a two-period model. The following proposition uses the implicit function theorem to calculate the derivative of the equity premium with respect to the intensity of the disposition effect.

Proposition 6

On the level curve $E\left[v(W_2,\lambda',\theta)\right]=0$ *the derivative of the equity premium with respect to* θ *is given by:*

$$
\frac{d\pi}{d\theta}=\frac{\pi(\pi+\sigma+r)}{r+(1-\theta)(\pi+\sigma)+(2-\theta)(r+\pi)+\lambda'\left(r+\pi-\sigma\right)}
$$

The risk premium required by an investor is then an increasing function of θ, as illustrated on figure 2. More over, the derivative in proposition 6 also increases in θ. It justifies the convex curve obtained on figure 2.

Figure 2 around here

5. Generalization : A multi-period simulation approach

We consider now a horizon T divided into N sub-periods. To keep things simple, we assume that, if a switch occurs, the investor's wealth remains invested in the risk-free asset up to date T. This assumption means that the investor engages in *narrow framing* or *mental accounting*[7]. She gets utility from the return of the investment in a single stock, even if this stock is included in a larger portfolio which is periodically rebalanced. Barberis and Huang (2001) show that a range of empirical phenomena may be explained by this individual stock accounting.

Let μ denote the continuous yearly expected return on the stock, σ the corresponding standard deviation and r_f the (continuous) risk-free rate. θ_L denotes the local probability of switching, that is the probability of switching in any given sub-period. The stock price process S is assumed to be a geometric Brownian motion with parameters (μ, σ) on the interval [0;T].

The investor regularly values her portfolio and may switch to bonds (with probability θ_L) at each intermediate date if the stock price is above the reference price. After n sub-periods, the reference price is $S_0 \exp\left(\frac{r_f n}{N}\right)$. We first simulate the increments of a general Brownian motion z with parameters (μ, σ).

We then build a path of the stock price process:

$$\begin{aligned} &S_0 = 1 \\ &S_{n/N} = S_{(n-1)/N} \exp\left(Y_n - Y_{n-1}\right) \end{aligned}$$

where $Y_n = z_{n/N}$.

The disposition effect is introduced in the following way. Let X be a uniform random variable on the interval [0;1] and W denotes the wealth process of the investor. Assume that investor's wealth is still invested in stocks at date n; we define:

[7] The idea of mental accounting was introduced by Richard Thaler (Thaler, 1980)

$W_{(n+1)/N} = If\left[(S_{n/N} > S_0 \exp(r_f n/N) \text{ and } X < \theta_L), W_{n/N} \exp\left(r_f n/N\right), W_{n/N} \exp\left(Y_{n+1} - Y_n\right)\right]$

This "computer-style" condition means that if $X < \theta_L$ the investor sells her stocks if the stock price is above the reference price ($S_{n/N} > S_0 \exp(r_f n/N)$). In this case, her wealth evolves at the risk-free rate and we get:

$$W_{(n+1)/N} = W_{n/N} \exp\left(r_f n/N\right)$$

If $S_{n/N} \leq S_0 \exp(r_f n/N)$ or $X \geq \theta_L$, the investor remains invested in stocks. We then obtain:

$$W_{(n+1)/N} = W_{n/N} \exp\left(Y_{n+1} - Y_n\right)$$

The link between θ_L, the local probability of switching, and θ, the probability of switching during the interval $[0;T]$, is complicated (due to the "no switching property" on loosing paths). No tractable formulation is available for θ. However, an upper and lower bound can be easily established by the following inequalities:

$$\frac{1}{2}\left(1-(1-\theta_L)^N\right) \leq \theta \leq 1-(1-\theta_L)^N$$

The upper bound is obtained by relaxing the assumption that no switching occurs when the stock price is too low since $(1-\theta_L)^N$ is the probability that a switch never happens in this case. The lower bound is linked to several assumptions on the price process. If the risk premium was equal to 0, the probability of being in the gain region would be equal to the one of being in the loss region (where no switching can occur). In other words, the probability of switching would be roughly one-half of the upper bound. However, when the risk premium is positive, the probability of switching is greater than this quantity.

Figure 3 around here

For example, with weekly returns (N = 52) and $\theta_L = 0.01$ we get $0.2 \leq \theta \leq 0.4$. Figure 3 illustrates these inequalities. The highest (lowest) curve show the upper (lower) bound for the probability of switching when the number of evaluation periods varies. The mid-curve reports the proportion of switching in simulations with 10 000 trials for each value of *N*.

The following simulations address three questions:

1. What is the relationship between θ_L and the moments of the yearly return obtained by the disposition investor?
2. Can we confirm in a multiperiod setting the results, obtained in the preceding section, about the link between the risk premium and the intensity of the disposition effect?
3. What is the effect of the evaluation period on the risk premium for the disposition investor?

5.1 The intensity of the disposition effect and the equity premium

Table 1 gives the results obtained when the local probability of switching (first column) varies from 0 to 0.1. We observe a decrease of about 80bps of the expected return when $\theta_L = 0.01$ (with respect to the situation without any disposition effect). Obviously, θ is a non-linear increasing function of θ_L.

Table 1 confirms the results obtained in the two-period model. The expected return and the standard deviation are decreasing functions of the probability of switching. Moreover, the Sharpe ratio (equal to the ratio of the expected return and the standard deviation because the risk-free rate is set equal to 0) is also globally decreasing and concave. However, when θ_L is high, we observe some instability in the evolution of the ratio around 0.23, due to the simulation process. In fact, the global proportion of switching is around 75 % when θ_L is greater than 0.07. The exact value of the ratio may be marginally influenced by the dates of switching. We also remark that the valuation function of our loss averse investor is a decreasing function of θ_L. In other words, the Sharpe ratio and the valuation function move in the same direction. It was not so intuitive because the Sharpe ratio is based on the variance of returns when the valuation function weights differently gains and losses.

Table 1 around here

Starting from a Gaussian distribution, the skewness becomes negative and the kurtosis increases when θ_L increases. To illustrate these results, Figure 4 reports the histogram of the distribution of returns over 10 000 paths when the weekly probability of switching is equal to 0.01.

Figure 4 around here

5.2 The role of the evaluation period

Benartzi and Thaler (1995) introduced the notion of *myopic loss aversion* to explain the equity premium. They suggested that even if an investor has a long horizon, she values her portfolio frequently and gets utility from wealth variations between two dates of valuation. They conclude that the historical equity premium may be explained with a 1-year valuation period, using reasonable parameters for the valuation function. In our framework, we can expect that the valuation period also plays a role. First, for a given θ_L, the global probability of switching increases with the number of valuation periods. If the disposition effect is seen like a psychological bias, it works at each evaluation date because the investor is prone to switch to bonds at each date she values her portfolio. Second, if we analyze the case of a given θ, switching appears at sooner dates when N increases. Consequently, we may expect that the expected return is lower when N is large for a given θ.

Table 2 reports the results obtained for a number of evaluation periods (from 10 to 300) on a horizon of 1 year. The parameter θ_L is equal to 0.01. Not surprisingly, θ increases when the evaluation period shortens; it leads to a decreasing expected return. When the investor evaluates her portfolio on an almost monthly basis ($N = 10$), the expected return is equal to 5.99 %, that is a value very close to the one obtained without a disposition effect. In this case, the global probability of switching is only 5 %. When a daily evaluation is considered, θ is equal to 65 % and the expected return falls to 3.7 %, that is a loss of 2.3 % with respect to the initial situation. Consequently, an investor knowing she is prone to the disposition effect will require a higher risk premium to invest in stocks. As a large majority of empirical studies show that investors (individual or professional) are victims of the disposition effect, it is reasonable to consider that many investors are conscious to be prone to this bias. It suggests that they require a greater risk premium to invest in stocks.

Table 2 around here

6. Concluding remarks

The model presented in this paper is simple and parsimonious because of the assumption of a piecewise linear valuation function. Only two parameters matter, the loss aversion coefficient and the probability of switching from stocks to bonds. The curvature of the valuation function, which we voluntarily neglect here, is often used as an argument to justify the disposition effect. In our analysis, the focus is placed on the influence of the consciousness of the disposition effect on the risk premium required by investors.

Most investors are prone to the disposition effect. Our purpose was not to say that every investor is conscious of this bias but that if a non negligible part of them is, it has consequences on the risk premium they require and, a fortiori, on the equity premium.

Further research is needed in several directions. First, we assumed that investors remain fully invested in bonds after a switch. This assumption could be relaxed but it is difficult to choose among all the possible alternative strategies.

A second direction could be to consider a continuous-time model, considering that the investor can switch at any moment. The advantage of a piecewise linear valuation function is that it can be written as the terminal payoff of a portfolio of call and put options. Switching from bonds to stocks before the horizon of investment is equivalent to an early exercise of the two types of options. Though the rules of optimal early exercise for a put option are well known, the corresponding rules for a portfolio of calls and puts are much more complicated and not already known. Moreover, switching to bonds may be interpreted as the exercise of a long position on call options and a short position on put options. But when you are short on options you don't hold the right to exercise. Consequently, this question, though interesting, is very difficult to address.

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Appendix

Proof of proposition 1

1) It is well known that the absence of arbitrage opportunities implies $d < r < u$. As mentioned before, indifference between the stock and the risk-free asset means:

$$E\left[v(S_1/r)\right]=\frac{1}{r}\left(p_2\left(uS_0-rS_0\right)-\lambda p_1\left(rS_0-dS_0\right)\right)=0 \qquad \text{(A1)}$$

Equation (2) follows immediately.

2) The expected gross return on the stock is equal to:

$$E\left[S_1 / S_0\right] = p_2 u + p_1 d$$

We also have:

$$\lambda^* > 1 \Leftrightarrow p_2(u-r) > p_1(r-d) \Leftrightarrow p_2 u + p_1 d > r$$

The result in point (2) is then obvious. ■

Proof of corollary 2

As the initial wealth is assumed to be 0, the agent borrows $\phi_0 + S_0$ at time 0, the put being devoted to protect the risky portfolio against returns lower than *r*; writing the expected valuation function gives the following relationship :

$$p_2\left(uS_0 - (S_0+\phi_0)r\right) - \lambda^*(1-p_2)\left[(S_0+\phi_0)r - dS_0 - (rS_0 - dS_0)\right] = 0$$

The first term corresponds to an up-state in which the put option is not exercised. The second term corresponds to a down-state and includes three values: the terminal value of the stock, the terminal payment of the put option and, finally, the reimbursement of the amount borrowed at date 0.

After elementary simplifications, we get:

$$p_2\left(uS_0 - (S_0+\phi_0)r\right) - \lambda^*(1-p_2)\phi_0 r = 0$$

The result announced in the corollary is immediately obtained by replacing λ^* by its value.■

Proof of proposition 3

To simplify the notations, we denote $p_2 = p$ and consider an initial price of the stock equal to 1. In fact, the sign of $E\left[v(S_2)\right]$ is independent of S_0. Using this simplifying assumption, the stock price takes three values at date 2, respectively u^2, ud and d^2 with probabilities p^2, $2p(1\text{-}p)$ and $(1-p)^2$.

Two cases must be considered, depending on the position of ud with respect to r^2. If $ud > r^2$ an up-down sequence generates a gain. On the contrary, if $ud < r^2$ the same sequence generates a loss.

1) $ud > r^2$

$$E\left[v(S_2)\right] = p^2(u^2 - r^2) + 2p(1-p)(ud - r^2) - \lambda^*(1-p)^2(r^2 - d^2)$$

Replacing λ^* by its value and dividing the result by *p* leads to:

$$\frac{E\left[v(S_2)\right]}{p} = p(u^2 - r^2) + 2(1-p)(ud - r^2) - (1-p)(r+d)(u-r)$$

A few lines of elementary calculations reduce this expression to:

$$\frac{E\left[v(S_2)\right]}{p} = (u+r)\left(p(u-r) + (1-p)(d-r)\right) \qquad \text{(A2)}$$

The second term in the RHS of (A2) is the excess return on the stock, relative to the risk-free rate. In proposition 1, we proved that it is positive as soon as the critical investor is loss averse (corresponding to $\lambda^* > 1$).

2) $ud < r^2$

$$\begin{aligned} E\left[v(S_2)\right] &= p^2(u^2 - r^2) - \lambda^*\left(2p(1-p)(r^2 - ud) + (1-p)^2(r^2 - d^2)\right) \\ &= p^2(u^2 - r^2) - p\frac{u-r}{r-d}\left(2p(r^2 - ud) + (1-p)(r^2 - d^2)\right) \\ &= p\frac{u-r}{r-d}\left[p(u+r)(r-d) - \left(2p(r^2 - ud) + (1-p)(r^2 - d^2)\right)\right] \end{aligned}$$

A few more transformations give:

$$E\left[v(S_2)\right] = p\frac{u-r}{r-d}(r+d)\left[p(u-d) - (r-d)\right]$$

We now use the point (2) of proposition 1, saying that the excess return on the stock is positive. It implies that:

$$p(u-d) - (r-d) > p(u-d) - (pu + (1-p)d - d) = 0$$

We then get $E\left[v(S_2)\right] > 0$.

Another way to get this result is to remark that a positive risk premium is equivalent to an inequality between the risk-neutral probability of an up-state and the corresponding real probability, the former being lower than the latter. This inequality is in fact:

$$p > \frac{r-d}{u-d} \Leftrightarrow p(u-d) - (r-d) > 0$$

Proof of proposition 4

1) We first analyze the case $ud > r^2$.

The expected valuation function of the one-period critical investor is worth:

$$E\left[v(W_2,\lambda^*,\theta)\right]=\left[p^2(1-\theta)(u^2-r^2)+p\theta(ur-r^2)+(2-\theta)p(1-p)(ud-r^2)\right]$$
$$-\lambda^*(1-p)^2(r^2-d^2)$$

where $\lambda^*=\frac{p}{1-p}\frac{u-r}{r-d}$. We then obtain :

$$\begin{aligned}E\left[v(W_2,\lambda^*,\theta)\right]&=p\left[p(1-\theta)(u^2-r^2)+\theta(ur-r^2)+(2-\theta)(1-p)(ud-r^2)\right]-\lambda^*(1-p)^2(r^2-d^2)\\&=p\left[p(1-\theta)(u^2-r^2)+\theta(ur-r^2)+(2-\theta)(1-p)(ud-r^2)\right]-\frac{u-r}{r-d}p(1-p)(r^2-d^2)\\&=p\left[p(1-\theta)(u^2-r^2)+\theta(ur-r^2)+(2-\theta)(1-p)(ud-r^2)-(1-p)(u-r)(r+d)\right]\\&=p\left[p(1-\theta)u^2+\theta ur+(2-\theta)(1-p)ud-(1-p)(ur+ud-rd)-r^2\right]\\&=p\left[p(1-\theta)u^2+\theta ur+(1-\theta)(1-p)ud-r(1-p)(u-d)-r^2\right]\end{aligned}$$

We now remark that:

$$\begin{aligned}\frac{\partial E\left[v(W_2,\lambda^*,\theta)\right]}{\partial\theta}&=p\left[-pu^2+ur-(1-p)ud\right]\\&=-pu\left[(1-p)d+pu-r\right]<0\end{aligned}$$

The derivative of the valuation function with respect to θ is negative as soon as the risk premium (which is the term between brackets) on the stock is positive.

We now calculate the valuation function at $\theta = 1$.

$$\begin{aligned}E\left[v(W_2,\lambda^*,1)\right]&=p\left[ur-r(1-p)(u-d)-r^2\right]\\&=p\left[r(u-r)-r(1-p)(u-d)\right]\\&=pr(u-d)\left[\frac{u-r}{u-d}-(1-p)\right]\end{aligned}$$

The term between brackets is the difference between the risk-neutral probability of a down-state and the corresponding actual probability. It is well-known that when the risk premium on the risky asset is positive, the risk-neutral probability of the up-state is lower than the corresponding real probability. It follows that the reverse inequality is satisfied by the probability of reaching the down-state. It follows that

$$E\left[v(W_2,\lambda^*,1)\right]>0$$

2) When $ud < r^2$ the expected value function is written as:

$$\begin{aligned}E\left[v(W_2,\lambda^*,\theta)\right]&=\left[p^2(1-\theta)(u^2-r^2)+\theta(ur-r^2)\right]\\&-\lambda^*\left[(2-\theta)p(1-p)(r^2-ud)+(1-p)^2(r^2-d^2)\right]\end{aligned}$$

because a up-down sequence now generates a loss.

Replacing λ^* by its value, and simplifying by $(1-p)$ leads to:

$$\begin{aligned}E\left[v(W_2,\lambda^*,\theta)\right]&=\left[p^2(1-\theta)(u^2-r^2)+p\theta(ur-r^2)\right]\\&-p\frac{u-r}{r-d}\left[(2-\theta)p(r^2-ud)+(1-p)(r^2-d^2)\right]\\&=p\left[p(1-\theta)(u^2-r^2)+\theta(ur-r^2)-\frac{u-r}{r-d}\left[(2-\theta)p(r^2-ud)+(1-p)(r^2-d^2)\right]\right]\end{aligned}$$

Taking the derivative with respect to θ gives:

$$\begin{aligned}\frac{\partial E\left[v(W_2,\lambda^*,\theta)\right]}{\partial\theta}&=p\left[-p(u^2-r^2)+(ur-r^2)+\frac{u-r}{r-d}p(r^2-ud)\right]\\&=p\left[-r^2(1-p-\frac{u-r}{r-d}p)-p(u^2)+(ur)-\frac{u-r}{r-d}p(ud)\right]\\&=p\left[(-r^2+ur)(1-p\frac{u-d}{r-d})\right]\\&=rp\left[(u-r)(1-p\frac{u-d}{r-d})\right]\\&=rp\left[\frac{u-r}{r-d}(r-d-p(u-d))\right]=rp\left[\frac{u-r}{r-d}(r-pu-(1-p)d)\right]\end{aligned}$$

Just as in point (1), the positive risk premium allows us to conclude that $\frac{\partial E\left[v(W_2,\lambda^*,\theta)\right]}{\partial\theta}<0$.

We now calculate the valuation function at $\theta = 1$.

$$\begin{aligned} E\left[v(W_2,\lambda^*,1)\right] &= p\left[(ur-r^2)-\frac{u-r}{r-d}\left[p(r^2-ud)+(1-p)(r^2-d^2)\right]\right] \\ &= p(u-r)\left[r-\frac{1}{r-d}\left[p(r^2-ud)+(1-p)(r^2-d^2)\right]\right] \\ &= p\frac{u-r}{r-d}\left[r^2-rd-p(r^2-ud)-(1-p)(r^2-d^2)\right] \\ &= p\frac{u-r}{r-d}\left[-rd+pud+(1-p)d^2\right] \\ &= pd\frac{u-r}{r-d}\left[pu+(1-p)d-r\right]>0 \end{aligned}$$

Proof of proposition 5

λ' depends on whether $ud-r^2$ is positive or negative. Consider first the case $ud-r^2>0$. We get for the first investor:

$$E\left[v(W_2,\lambda',0)\right]=\frac{1}{4}\left[(u^2-r^2)+2(ud-r^2)+\lambda'(d^2-r^2)\right]=0$$

We deduce:

$$\lambda'=\frac{(u^2-r^2)+2(ud-r^2)}{(r^2-d^2)}=\frac{\left[(r+\pi+\sigma)^2-r^2+2((r+\pi+\sigma)(r+\pi-\sigma)-r^2)\right]}{(r^2-(r+\pi-\sigma)^2)}$$

After elementary simplifications, we obtain the desired result:

$$\lambda'=\frac{4\pi^2-(\sigma-\pi)^2+2r(3\pi+\sigma)}{(\sigma-\pi)(2r+\pi-\sigma)}$$

Consider now the case $ud-r^2<0$ (even if it doesn't correspond to the historical value of the parameters). The valuation function is worth:

$$E\left[v(W_2)\right]=\frac{1}{4}\left[(u^2-r^2)+\lambda'\left(2ud+d^2-3r^2\right)\right]$$

We deduce the critical loss aversion parameter:

$$\lambda'=\frac{u^2-r^2}{3r^2-2ud-d^2}$$

Replacing *u*, *r* and *d* by their values we get, after a few lines of calculations:

$$\lambda' = \frac{4\sigma(r+\pi)}{(\sigma+\pi)^2 - 4\pi^2 + 2r(\sigma - 3\pi)}$$

Proof of proposition 6

$$\begin{aligned}
E\left[v\right] = 0.25(&(1-\theta)((r+\pi+\sigma)^2 - r^2) + 2\theta((r+\pi+\sigma)r - r^2) \\
&+(2-\theta)((r+\pi+\sigma)(r+\pi-\sigma) - r^2) \\
&+\lambda'\left((r+\pi-\sigma)(r+\pi-\sigma) - r^2\right)) \\
&= 0.25((1-\theta)((\pi+\sigma)^2 + 2r(\pi+\sigma)) + 2\theta r(\pi+\sigma) \\
&+(2-\theta)(\pi^2 - \sigma^2 + 2r\pi) \\
&+\lambda'\left((\pi-\sigma)^2 + 2r(\pi-\sigma)\right))
\end{aligned}$$

We then get:

$$\begin{aligned}
\frac{\partial E\left[v\right]}{\partial\theta} &= -0.25(((\pi+\sigma)^2 + 2r(\pi+\sigma)) + 2r(\pi+\sigma) \\
&\quad -(\pi^2 - \sigma^2 + 2r\pi)) \\
&= -0.25((\pi+\sigma)^2 + (\pi^2 - \sigma^2 + 2r\pi)) \\
&= -0.5\pi(\pi+\sigma+r)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial E\left[v\right]}{\partial\pi} &= 0.25(2(1-\theta)(r+\pi+\sigma) + 2r\theta + 2(2-\theta)(r+\pi) \\
&\quad +2\lambda'\left(r+\pi-\sigma\right)) \\
&= 0.5((1-\theta)(r+\pi+\sigma) + r\theta + (2-\theta)(r+\pi) \\
&\quad +\lambda'\left(r+\pi-\sigma\right))
\end{aligned}$$

Finally, the implicit function theorem leads to:

$$\frac{d\pi}{d\theta} = \frac{\pi(\pi+\sigma+r)}{(1-\theta)(r+\pi+\sigma) + r\theta + (2-\theta)(r+\pi) + \lambda'\left(r+\pi-\sigma\right)}$$

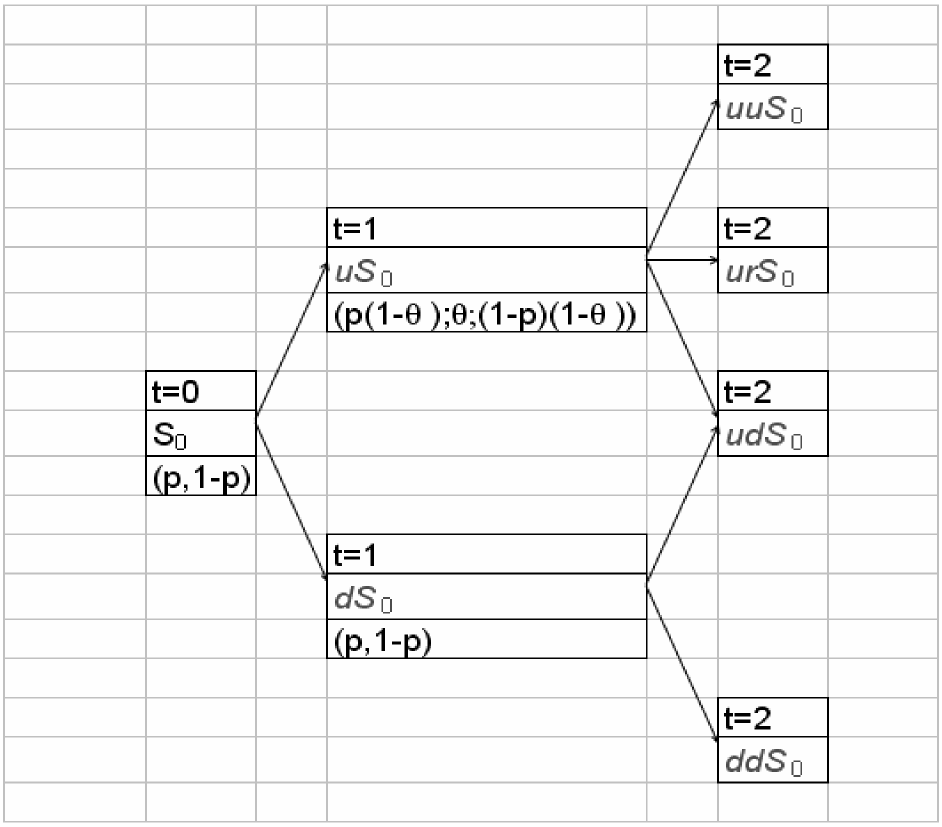

Figure 1: The two-period model when the disposition effect is introduced

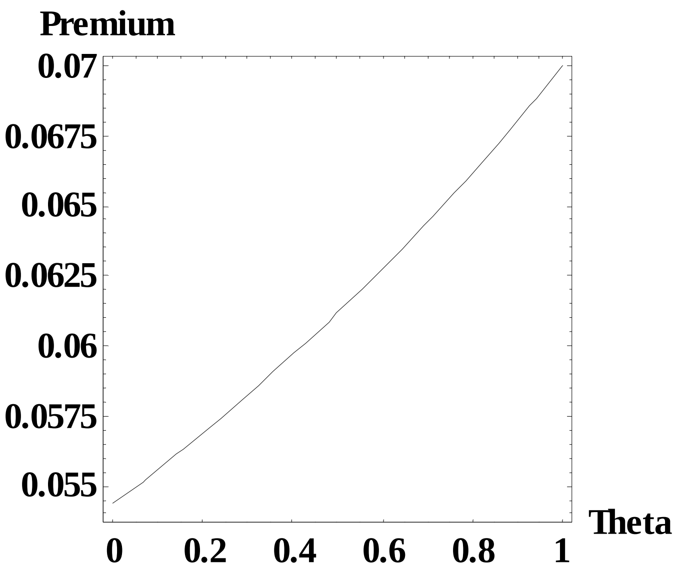

Figure 2: The risk premium as a function of θ

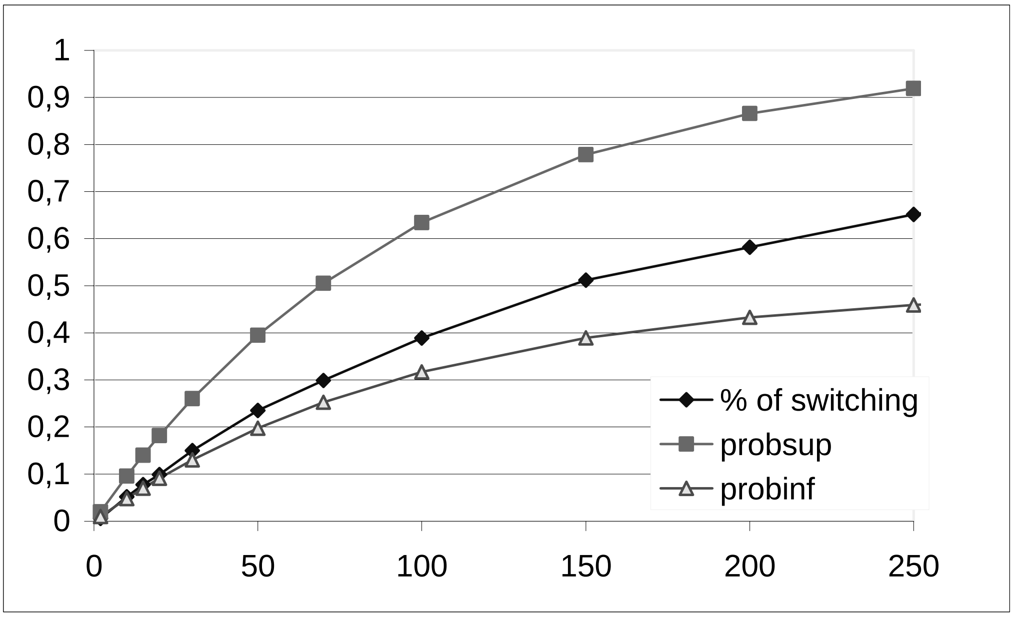

Figure 3: Upper (probsup) and lower (probinf) bound for the probability of switching

$$r = 0; \mu = 6\%; \sigma = 20\%; \theta = 0.01$$

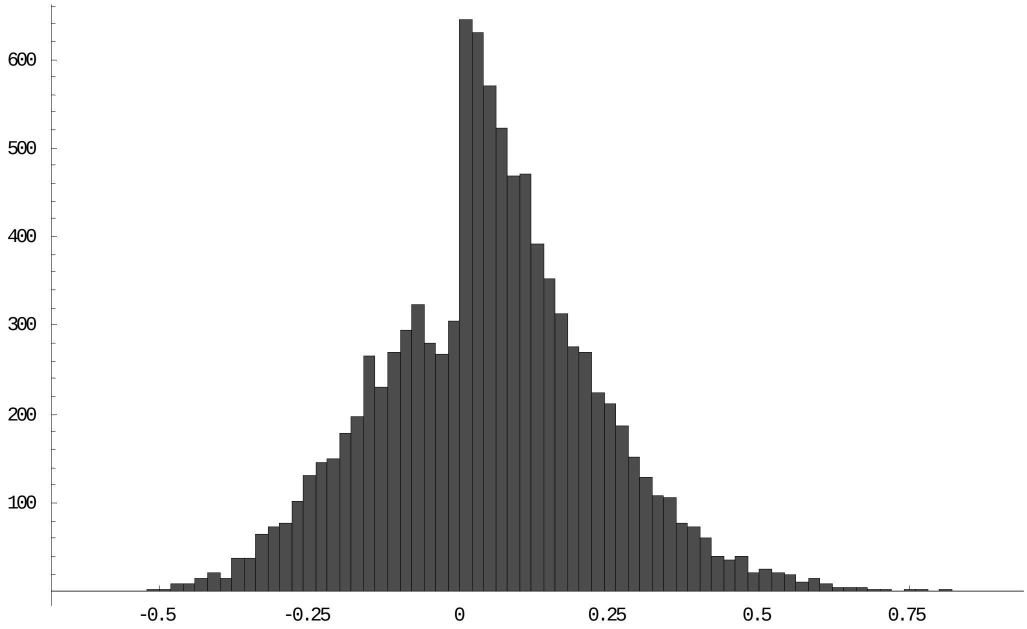

Figure 4: Simulation of returns (10 000 draws) for 52 weeks with

$$r = 0; \mu = 6\%; \sigma = 20\%; \theta_L = 0.01$$

Table 1: Simulation of returns (10 000 draws) for 52 weeks

$$r = 0; \mu = 6\%; \sigma = 20\%$$

θ_L	% of switching	Expected return	Standard deviation	Skewness	Kurtosis	Sharpe ratio	Valuation function
0	0.00	5.99 %	0.199	0.246	3.07	0.301	0.0246
0.01	0.238	5.18 %	0.183	0.138	3.29	0.282	0.0167
0.02	0.411	4.59 %	0.169	0.04	3.81	0.271	0.0126
0.03	0.5199	4.11 %	0.159	-0.09	4.15	0.258	0.0085
0.04	0.5901	3.69 %	0.149	-0.21	4.30	0.246	0.0047
0.05	0.6705	3.65 %	0.139	-0.376	4.75	0.261	0.075
0.06	0.7075	3.20 %	0.135	-0.519	5.02	0.236	0.002
0.07	0.7382	2.92 %	0.127	-0.800	5.33	0.230	0.0008
0.08	0.7526	2.86 %	0.123	-0.749	5.65	0.231	0.0014
0.09	0.7708	2.56 %	0.119	-0.96	5.94	0.214	-0.001
0.1	0.7992	2.73 %	0.114	-1.06	6.34	0.239	0.002

Table 2: Simulation of returns (10 000 draws) for 52 weeks

$$r = 0; \mu = 6\%; \sigma = 20\%; \theta_L = 0.01$$

N	% of switching	Expected return	Standard deviation	Skewness	Kurtosis	Sharpe ratio	Valuation function
10	0.0517	5.99 %	0.1984	-0.005	3.06	0.301	0.024
15	0.0777	5.65 %	0.194	-0.05	3.14	0.291	0.02
20	0.099	5.75 %	0.193	-0.27	3.16	0.296	0.0219
30	0.15	5.40 %	0.190	0.066	3.21	0.282	0.0180
50	0.2353	5.21 %	0.183	0.112	3.34	0.284	0.0172
70	0.299	4.85 %	0.178	0.094	3.48	0.272	0.0133
100	0.389	4.75 %	0.173	0.051	3.69	0.273	0.0137
150	0.512	4.14 %	0.163	-0.049	3.97	0.253	0.008
200	0.582	3.53 %	0.154	-0.21	4.48	0.228	0.002
250	0.6515	3.70 %	0.147	-0.299	4.83	0.251	0.0069
300	0.6998	3.52 %	0.14	-0.462	4.73	0.252	0.0061

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