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# How to solve the St Petersburg Paradox in Rank-Dependent Models?

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**Abstract**

The Cumulative Prospect Theory, as it was specified by Tversky and Kahneman (1992) does not explain the St Petersburg Paradox. This study shows that the solutions proposed in the literature (Blavatskky, 2005; Rieger and Wang, 2006) to guarantee, under rank dependant models, finite subjective utilities for any prospects with finite expected values have to cope with many limitations. In that framework, CPT fails to accommodate both gambling and insurance behavior. We suggested to replace the weighting function generally proposed in the literature with another specification which respects the following properties. 1) In order to guarantee finite subjective values for all prospects with finite expected values, the slope at zero should be finite. 2)To account for the fourfold pattern of risk attitudes, the probability weighting should be strong enough to overcome the concavity of the value function.

**Keywords** : St Petersburg Paradox, Cumulative Prospect Theory, Probability Weighting, Gambling.

**JEL Classification** : D81, C01.

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# 1 Introduction

The St. Petersburg Paradox posed in 1713 by Nicholas Bernoulli shows that for prospects with infinite expected monetary value decision makers are not willing to pay an infinite sum of money. This observation can be taken as an evidence against expected value theory. In fact, according to this theory introduced by Blaise Pascal, individuals evaluate risky prospects by their expected value. So any decision-maker should accept to pay an infinite amount of money for prospects with infinite expected value. In 1738, Daniel Bernoulli solved the paradox by introducing the idea of diminishing marginal utility. He postulated that individuals valuate prospects not by their expected value but by their expected utility where utility is not linearly related to outcomes but increases at a decreasing rate. So if we consider that individuals preferences are represented by a strictly increasing and concave utility function this paradox can be solved. Since this date, the expected utility (EU) theory was considered (for many years) as a benchmark for describing decision making under risk.

However, the Allais paradox (1953) and many other experimental studies (Slovic et Lichtenstein, 1968; Kahneman and Tversky, 1979) report persistent violations of EU theory. Moreover its predictions of behavior are seriously questioned. On the one hand, individuals preferences for insurance lead to a risk-averse behavior. On the other hand, acceptance of gambling indicates risk-seeking behavior. Two conflicting behavioral choices are therefore observed. The cumulative prospect theory (CPT) developed by Tversky and Kahneman (1992) was then proposed as an alternative model to the well established expected utility model. This theory is based on 4 important features: 1) Utility is defined over gains and losses rather than over final asset position. So risky prospects are evaluated relatively to a reference point. This reference point corresponds to the asset position one expects to reach. 2) The sensitivity relatively to the reference point is decreasing. The value function is then concave for gains and convex for losses. 3) Individuals have asymmetric perception of gains and losses: they are loss-adverse, hence the value function is steeper for losses than for gains. 4) Individuals do not use objective probabilities when evaluating risky prospects. They transform objective probabilities via a weighting function. They overweight the small probabilities of extreme outcomes (events at the upper tail of the distribution). Conversely, they underweight outcomes with average probabilities.

CPT stands for one of the most well-accepted alternatives to expected utility theory because its predictions of behavior are consistent with the recently accumulated empirical evidence on individual preferences. Although this model overwhelmed the EU predictions of behavior, it must cope with other types of difficulties. Blavatskky (2005) emphasized an intrinsic limitation of that model. The overweighting of small probabilities can lead to the re-occurrence of the St Petersburg Paradox. He showed that the valuation of a prospect (the subjective utility) by cumulative prospect theory can be infinite. However it is important to notice that this dilemma is not specific to rank dependent models such as CPT. It is now well known that even in expected utility framework the Bernoulli's resolution of the paradox is unsatisfactory. Actually, in EU framework, that is to say when individuals do not transform probabilities, the introduction of a concave utility function can resolve the paradox as it was introduced by N. Bernoulli. But the game can be modified (by making prizes grow sufficiently fast) so that the concavity of the utility function is not sufficient to guarantee a finite expected utility value[1]. Arrow proposed to resolve this problem by only considering distributions with finite expected value. In that case, the concavity of the utility function is sufficient to guarantee, under EU framework, a finite valuation. This assumption is quite realistic because none individuals or organizations can offer a prospect with infinite expected value. However, we cannot implement it under CPT framework. In fact, in cumulative prospect theory, a prospect with finite expected value can have an infinite subjective value (Rieger and Wang, 2006).

The purpose of the present paper is to determine how we can solve this paradox. Blavatskky (2005) and Rieger and Wang (2006) have already proposed some solutions to guarantee finite subjective values for all prospects with finite expected value. In this study, we explore the behavioral implications of these propositions. We establish that if we take them into account, the modified cumulative prospect theory is not anymore consistent with some behavior observed on the market. For example, there are situations where the CPT modified by these propositions cannot anymore accommodate both gambling and insurance behavior.

The paper is structured as follow: Section II reviews the cumulative prospect theory model paying particular attention to the functional form of the value and weighting function. In section III, we present how the paradox occurs under CPT and report the solutions derived by Blavatskyy (2005) and Rieger and Wang (2006) to resolve it. Section IV analyses the behavioral implications of these propositions. In section V, we propose a more appropriate solution to this paradox. Section VI concludes with a summary of our findings.

## 2 Cumulative Prospect Theory

In this section we briefly present the cumulative prospect theory formalized by Tversky and Kahneman in 1992. In a first version (1979) Kahneman and Tversky supposed that decision makers transform individual probabilities directly via a weighting function. This

[1]This super paradox was first illustrated by Menger (1934).

assumption leads to preferences that violate the first order stochastic dominance criteria. In the cumulative version, they took into account Quiggin and Yaari's work and applied the probability weighting function to the cumulative probability distribution. Therefore, while attitude towards risk is fully characterized by the value function under expected utility theory, under cumulative prospect theory, attitude towards risk is determined simultaneously by the value function and the cumulative weighing function.

Consider a prospect $X$ defined by:

$$X = ((x_i, p_i)i = -m, ....n)$$

with $x_{-m} < x_{-m+1} < .... < x_0 = 0 < x_1 < x_2 < ... < x_n$.

We have mentioned previously that gains and losses are evaluated differently by individuals. In order to take into account this assumption, the evaluation function $V$ of a prospect $X$ is defined by:

$$V(X) = V(X^+) + V(X^-)$$

where $X^+ = \max(X;0)$ et $X^- = \min(X;0)$.

We set:

$$V(X^+) = \sum_{i=0}^{n} \pi_i^+ v(x_i)$$
$$V(X^-) = \sum_{i=-m}^{0} \pi_i^- v(x_i)$$

where $v$ is a strictly increasing value function defined with respect to a reference point satisfying $v(x_0) = v(0) = 0$.

$\pi^+ = (\pi_0^+, ....\pi_n^+)$ and $\pi^- = (\pi_{-m}^-, , ....\pi_0^-)$ are the weighting functions for gains and losses respectively defined by:

$$\begin{aligned}
\pi_n^+ &= w^+(p_n) \\
\pi_{-m}^- &= w^-(p_{-m}) \\
\pi_i^+ &= w^+(p_i + ..... + p_n) - w^+(p_{i+1} + ..... + p_n) \ \text{ with } \ 0 \leq \text{i} \leq \text{n-1} \\
\pi_i^- &= w^-(p_{-m} + ..... + p_i) - w^-(p_{-m} + ..... + p_{i-1}) \ \text{ with} - m \leq i \leq 0
\end{aligned}$$

with $w^+(0) = 0 = w^-(0)$ and $w^+(1) = 1 = w^-(1)$

Consider $F$ the cumulative distribution function of $X$. We notice that $w^+(p_i)$ is applied to the decumulative distribution function of $X$ (i.e.))

$$w^+(\sum_{j=1}^{n} p_j) = w^+(1 - F(x_{i-1}))$$

whereas $w^-(p_i)$ is applied to the cumulative distribution function of $X$

$$w^-(\sum_{j=-m}^{i} p_j) = w^-(F(x_i))$$

Tversky and Kahneman (1992) proposed the following functional form for the value function:

$$v(x) = \begin{Bmatrix} x^\alpha & \text{if } x > 0 \\ -\lambda(-x)^{-\beta} & \text{if } \ x < 0 \end{Bmatrix}$$

For $0 < \alpha < 1$ and $0 < \beta < 1$ the value function $v$ is concave over gains and convex over losses. It is kinked at the origin and steeper for losses than for gains. The parameter $\lambda$ describes the degree of loss aversion Köbberling and Wakker (2005). Based on experimental evidences, Tversky and Kahneman estimated the values of the parameters $\alpha$, $\beta$, and $\lambda$ $\alpha = \beta = 0.88$ and $\lambda = 2.25$.

They proposed the following functional form for the weighting function:

$$w^+(p) = \frac{p^{\gamma+}}{[p^{\gamma+} + (1-p)^{\gamma+}]^{1/\gamma+}} \qquad w^-(p) = \frac{p^{\gamma-}}{[p^{\gamma-} + (1-p)^{\gamma-}]^{1/\gamma-}}$$

For $\gamma < 1$, this functional form integrates the overweighting of low probabilities and the greater sensitivity for changes in probabilities for extremely low and extremely high probabilities. The weighting function is concave near 0 and convex near 1. Tversky and Kahneman estimated the parameters $\gamma^+$ and $\gamma^-$ as 0.61 and 0.69.

# 3 The St Petersburg Paradox

The St Petersburg Paradox is usually explained by the following example: Let's consider a gamble $L$ in which the player gets $2^n$ euros when the coins lands heads for the first time at the $n^{th}$ throw. $L$ has an infinite expected value.

$$E(L) = \tfrac{1}{2} \times 2 + (\tfrac{1}{2})^2 \times 2^2 + ........ + (\tfrac{1}{2})^n \times 2^n + ...... = \sum_{k=1}^{\infty} 1 = +\infty \qquad (1)$$

According to number of experiments, the maximum price an individual is willing to pay for this gamble is around 3 euros. This observation can be taken as an evidence against expected value theory. Daniel Bernoulli (1738) proposed to replace in (1) the monetary value of each outcome with their subjective utility (where the subjective utilities are represented by a strictly concave utility function). In that framework the strictly concave utility function $u(x) = \ln x$ leads to:

$$UE(L) = \sum_{k=1}^{\infty} (\tfrac{1}{2})^k \times \ln(2^k) = 2\ln 2 < +\infty \qquad (2)$$

This solution permits to solve this particular paradox. Since this resolution, expected utility theory became, for more than 200 years, the major model of choices under risk. However, empirical evidences have recently showed that EU theory fails to provide a good explanation of individual behavior under risk. These observations have motivated the development of alternative models of choices. One of the most famous is the cumulative prospect theory (presented in the previous section) developed by Tversky and Kahneman (1992).

Even if this model is deemed to be one of the best alternatives to EU theory, it also has to deal with some difficulties. Actually, Blavatskky (2005) established that, under CPT, the overweighting of small probabilities restores the St Petersburg Paradox. Under CPT, the subjective utility $(V)$ of the game described above $(L)$ is given by:

$$\begin{aligned} V(L) &= \sum_{n=1}^{+\infty} v(2^n) \times \left[ w(\sum_{i=n}^{+\infty} 2^{-i}) - w(\sum_{i=n+1}^{+\infty} 2^{-i}) \right] \\ &= \sum_{n=1}^{+\infty} v(2^n) \times \left[ w(2^{1-n}) - w(2^{-n}) \right] \qquad (3) \end{aligned}$$

In section 2, we underline that the functional forms proposed for $v$ and $w$ by Tversky and Kahneman are $v(x) = x^{\alpha}$ with $\alpha > 0$ and $w(p) = p^{\gamma}/(p^{\gamma} + (1-p)^{\gamma})^{1/\gamma}$ with $0 < \gamma < 1$.

As $\lim_{n \longrightarrow +\infty} 2^{1-n} = 0$ and $\lim_{n \longrightarrow +\infty} 2^{1-n} = 0$, $[p^\gamma + (1-p)^\gamma]^{1/\gamma}$converges to unity. In that case, the function $w$, specified by Tversky and Kahneman, can then be approximated by $p^\gamma$. According to these elements we can rewrite $V(L)$ as:

$$\begin{aligned} V(L) &\approx (2^\gamma - 1)\sum_{n=1}^{+\infty} 2^{(\alpha-\gamma)n} \\ &\approx 0,526 \times \sum_{n=1}^{+\infty} 2^{0,27n} \longrightarrow +\infty \end{aligned} \tag{4}$$

One can object that this paradox does not involve a real problem insofar as the expected value of this game is infinite. Actually, it is not realist to assume that an institution can offer a prospect with unlimited expected value (Arrow, 1974). However, Pfiffelmann (2007) and Rieger and Wang (2006) pointed out that, under CPT, a prospect with finite expected value can have infinite subjective utility. Such a result is possible because the weighting function has an infinite slope at zero. Therefore, the more the probability is low, the more the overweighting is important. An extremely small probability can thus be infinitely overweighted. As the value function is unbounded, there are situations for which the subjective value of a consequence weighted by its decision weight can be infinitely high.

Rieger et Wang (2006) characterized situations where this problem can be resolved. They focused on fitting parameterized functional forms to CPT's functions and determined for which parameter combinations the model implies finite subjective value for all lotteries with finite expected value.

Let's consider $V$ the subjective utility under CPT[2]:

$$V(p) = \int_{-\infty}^{0} v(x)\frac{d}{dx}(w_{-}(F(x)))dx + \int_{0}^{+\infty} v(x)\frac{d}{dx}(w_{-}(F(x)))dx \tag{5}$$

where the value function $v$ is continuous, monotone, convex for $x < 0$ and concave for $x > 0$. Assume that there exists constants $\alpha$, $\beta > 0$ such that:

$$\begin{aligned} \lim_{x \longrightarrow +\infty} \frac{v(x)}{x^\alpha} &= v_1 \quad \in (0, +\infty) \\ \lim_{x \longrightarrow -\infty} \frac{|v(x)|}{|x|^\beta} &= v_2 \quad \in (0, +\infty) \end{aligned}$$

[2]For more details on the demonstration see Rieger and Wand (2006).

Assume that the weighting functions $w$ are continuous and strictly increasing from [0,1] to [0,1] such that $w(0) = 0$ and $w(1) = 1$. Moreover, assume that $w$ is continuously differentiable on ]0,1[ and that there are constants $\gamma^+$, $\gamma^-$ such that:

$$\begin{aligned}
\lim_{y \longrightarrow 0} \frac{w'_-(y)}{y^{\gamma^- -1}} &= w_1 \quad \in (0, +\infty) \\
\lim_{y \longrightarrow 1} \frac{1 - w'_+(y)}{(1-y)^{\gamma^+ -1}} &= w_2 \quad \in (0, +\infty)
\end{aligned}$$

Consider $p$ a probability distribution for which $E(p) < \infty$. If all the conditions described above are satisfied $V(p)$ is finite if $\alpha < \gamma^+$ et $\beta < \gamma^-$.

Thus if we consider the Tversky and Kahneman's specification for the value and weighting functions, the valuation of any prospect by CPT will be finite only if $\alpha < \gamma^+$ and $\beta < \gamma^-$. The estimates of $\alpha$, $\beta$, $\gamma^+$and $\gamma^-$ are usually obtained from parametric fitting to experimental data. The estimated parameters realized by Camerer and Ho (1994) and Wu and Gonzales (1996) are the only one that are consistent with these conditions. In that case, the concavity of the value function is sufficiently strong relative to the probability weighting function to avoid the St Petersburg Paradox.

Rieger and Wang (2006) proposed another solution to avoid the paradox under CPT. They suggested to consider a polynomial of degree three as a weighting function. The specification is given by:

$$w(p) = \frac{3 - 3b}{a^2 - a + 1} \times (p^3 - (a+1)p^2 + ap) + p \qquad (6)$$

with $a \in (0,1)$ et $b \in (0,1)$.

As its slope at zero is finite, this weighting function permits to avoid infinite subjective utilities for all prospects with finite expected value.

# 4 The behavioral implications of the solutions proposed in the literature

If we take into account the propositions described above, the St Petersburg Paradox will not occur under CPT. But at the same time, this theory will loose a major part of its descriptive power.

## 4.1 The Camerer and Ho's (1994) and Wu and Gonzales' (1996) estimates

Rieger and Wang (2006) established that prospects with finite expected value will not have infinite subjective utility if the power coefficient of the value function is lower than the power coefficient of the probability weighting function. Two parameterized versions of CPT are consistent with this condition (Camerer and Ho, 1994; Wu and Gonzales, 1996). Accepting them will solve the paradox. However, they generate other kind of difficulties. Actually, these parameterizations of CPT cannot anymore accommodate the four-fold pattern of risk attitude (risk aversion for most gains and low probability losses, and risk seeking for most losses and low probability gains). For example, these versions of CPT fail to capture the gambling behavior observed on the market and more precisely the tendency of individuals to bet on unlikely gains (Neilson et Stowe, 2002). This pattern is still one of the most fundamental contributions of CPT as it was developed by Tversky and Kahneman. But it can emerge only if the probability weighting over-ride the curvature of the value function (for low probabilities). In fact, in rank dependant models, gambling behavior can be captured only if the overweighting of probabilities is strong enough to compensate the concavity of the value function. When $\alpha$ is lower than $\gamma$[3], the convexity of the weighting function cannot overcome the concavity of the value function. In that case, optimism generated by the weighting function does not offset risk aversion resulting from the value function. It is thus impossible to account for gambling behavior[4]. If we consider a value function whose power coefficient is lower than the coefficient of the probability weighting function (as it is suggested by Rieger and Wang), CPT does not restore the St Petersburg Paradox but in return it does not provide a good description of individual behavior under risk.

[3]For all usual estimates of $\gamma$.

[4]This fact is illustrated in appendice. We show that with the Camerer and Ho's and Wu and Gonzales' estimates the choice behavior under CPT is not consistent with the very famous lottery games euromillions.

## 4.2 The Rieger and Wang's polynomial function

### 4.2.1 Limitations of the specification

We underline previously that considering a polynomial weighting function instead of the Tversky and Kahneman's specification can solve the paradox. The functional form proposed by Rieger and Wang is given by:

$$w(p) = \frac{3-3b}{a^2-a+1} \times (p^3 - (a+1)p^2 + ap) + p$$

with $a \in (0,1)$ et $b \in (0,1)$.

As its slope at zero and unity is finite, the subjective utility of any finite expected value prospects will be finite. However, as the solution proposed above, the behavioral implication of this new specification of CPT does not allow for betting on unlikely gains. The highest slope of this function at zero (when $b = 0$ and $a = 1$) is actually equal to 4, which is too low. The small probabilities are then not enough overweighted. Therefore the model modified by the Rieger and Wang's weighting function do not imply that individuals insure against unlikely losses or bet on unlikely gains. In order to illustrate this fact, we apply CPT (the version proposed by Tversky and Kahneman and the one modified by Rieger and Wang) to the most popular European lottery: *Euromillion*. We obtain that with the modification operated by Rieger and Wang, CPT cannot anymore explain the popularity of this very famous game.

### 4.2.2 Illustration: the case of Euromillions

Euromillions is a unique lottery game played every Friday by hundreds of millions of players throughout Europe. One can play by using a playslip that contains six sets of main boards and lucky star boards. Each player selects five numbers on a main board and two numbers on the associated lucky start board to make one entry. A Euromillions lottery ticket costs euro 2. 50% of the money paid for a ticket goes directly to the operator selling it (in France euro 1 of every euro 2 ticket a player buy will go to the "Francaise des Jeux"). The remaining 50% of ticket fees goes into the "Common Prize Fund" out of which all prizes are paid. Table 1 represents the percentage share of the fund allocated to each prize with the corresponding probabilities[5].

We apply the two versions of CPT (the original and the one developed by Rieger and Wang) to this European game. In order to determine the average monetary game

[5] We assume that all the booster fund is devoted to the first rank.

Table 1: Euromillions

| gains rank | % Prize Fund | Probability |
|---|---|---|
| $1^{st}$ rank | 32%+6% | $\frac{1}{76\ 275\ 360}$ |
| $2^{nd}$ rank | 7.4% | $\frac{1}{5\ 448\ 240}$ |
| $3^{th}$ rank | 2.1% | $\frac{1}{3\ 632\ 160}$ |
| $4^{th}$ rank | 1.5% | $\frac{1}{339\ 002}$ |
| $5^{th}$ rank | 1% | $\frac{1}{24\ 214}$ |
| $6^{th}$ rank | 0.7% | $\frac{1}{16\ 143}$ |
| $7^{th}$ rank | 1% | $\frac{1}{7\ 705}$ |
| $8^{th}$ rank | 5.1% | $\frac{1}{550}$ |
| $9^{th}$ rank | 4.4% | $\frac{1}{538}$ |
| $10^{th}$ rank | 4.7% | $\frac{1}{367}$ |
| $11^{th}$ rank | 10.1% | $\frac{1}{102}$ |
| $12^{th}$ rank | 24% | $\frac{1}{38}$ |

for each winner at each rank, we assume a number of participants equal to 40 millions (so a prize fund also equal to 40 millions)[6]. The probability to win at the second rank is $\frac{1}{5\ 448\ 240}$, so on average there are 7 winners at this rank. The average gain per winner is then equal to: $\frac{40\ 000\ 000 \times 7.4\%}{7} = 422\ 857.14$. Table 2 displays the results of the game's valuation by CPT and CPT modified[7].

Table 2: Euromillions Valuation

| gains rank | Probabilities | Gains | $v(x_i)$ | $\pi_i$ CPT | $\pi_i$ CPT modified |
|---|---|---|---|---|---|
| $1^{st}$ rank | $1.311\times10^{-8}$ | 15 200 000 | 2 089 403.44 | $1.55501\times10^{-5}$ | $2.34607\times10^{-8}$ |
| $2^{nd}$ rank | $1.835\times10^{-7}$ | 422 857.14 | 89 340.55 | $6.55653\times10^{-5}$ | $3.2845\times10^{-7}$ |
| $3^{th}$ rank | $2.753\times10^{-7}$ | 76 363.63 | 19 012.08 | $5.72386\times10^{-5}$ | $4.92674\times10^{-7}$ |
| $4^{th}$ rank | $2.949\times10^{-6}$ | 5 128.2 | 1839.16 | 0.00032463 | $5.27862\times10^{-6}$ |
| $5^{th}$ rank | $4.129\times10^{-5}$ | 242.27 | 124.46 | 0.00175136 | $7.38969\times10^{-5}$ |
| $6^{th}$ rank | $6.194\times10^{-5}$ | 113.03 | 63.09 | 0.00153935 | 0.000110825 |
| $7^{th}$ rank | 0.0001297 | 77.05 | 44.7 | 0.00232386 | 0.000232125 |
| $8^{th}$ rank | 0.0018181 | 28.05 | 17.61 | 0.0160782 | 0.003242095 |
| $9^{th}$ rank | 0.0018587 | 23.67 | 14.98 | 0.0101514 | 0.003295609 |
| $10^{th}$ rank | 0.0027247 | 17.25 | 10.99 | 0.0115014 | 0.004796966 |
| $11^{th}$ rank | 0.0098039 | 10.30 | 6.44 | 0.0290591 | 0.016926816 |
| $12^{th}$ rank | 0.0263157 | 9.12 | 5.62 | 0.048556 | 0.042932219 |
| No gain | 0.9572 | 0 | -4.14 | 0.86336 | 0.912226886 |
| $\mathbf{V(X)}$ | | | | **37.94** | **-3.14** |

The valuation of the game with cumulative prospect theory, as it was developed by Tversky and Kahneman, is positive. But if we substitute the inverse S- shape probability

[6]The prize fund reached euro 38 734 739 the 12 October 2007, and euro 50 916 665 the 20 September.
[7]We consider a reference point of euro 2.

weighting function specified by Tversky and Kahneman with the one proposed by Rieger and Wang[8] the subjective utility becomes negative. Decision makers who transform probabilities via the polynomial weighting function would prefer keeping the price of the lottery ticket rather than participate in the lottery. This new version of CPT is therefore not able to explain the popularity of public lotteries. This limitation is quite severe, since betting on unlikely gains is one of the most important stylized facts that cumulative prospect theory aims to predict.

# 5 An alternative weighting function

Section III underlines that CPT should be remodeled if we want to apply it to problems of choices. The occurrence of the paradoxe under CPT comes from the overweighting of small probabilities. As the slope of the weighting function at zero is infinity, an extremely small probability can be infinitely overweighted. And as the slope of the value function do not decrease for the high values of outcomes, the subjective value of a consequence weighted by its decision weight can be infinitely high. In order to overcome this difficulty, we propose an alternative weighting function that avoids infinite values for subjective utility. This function should respect the following properties:

1. $w(0) = 0$ and $w(1) = 1$
2. $w$ strictly increasing on $[0, 1]$.
3. $w$ continuously differentiable on $[0, 1]$, with $w'(0)$ et $w'(1) \neq \infty$. We underlined previously that if the slope at zero is too low, CPT cannot accommodate the gambling and insurance behavior observed on the market. Therefore $w'(0)$ should be sufficiently strong.

## 5.1 A polynomial specification

Before attempting to build a new weighting function, it is important to clarify a point. The evidences presented previously criticize the way probabilities are transformed near 0 and 1 with the weighting function proposed by Tversky and Kahneman. These evidences challenge the specification of the function but not the experimental data Tversky and Kahneman have obtained. In fact, the problem does not concern the individual preferences they have elicited but the functional form of the function. We agree (and there is no doubt about that) that an inverse S shape weighting function, first concave then convex really

[8] In table 2, we took for the computation of $\pi$: $a = 0.4$ and $b = 0.5$. Nevertheless, the same result (a negative valuation) is obtained with any other combinations of $a$ and $b$.

represents the way individuals transform probabilities. The difficulties related to the use of the Tversky and Kahneman's weighting function only concerns its slope at 0 and 1. To overcome them, we consider a polynomial weighting function because the slope of this kind of specifications is always finite within the interval considered. As we do not challenge the experimental results Tversky and Kahneman have obtained, we estimate the coefficients of the polynomial from the points of the functional they have proposed. We remind that the slope at zero must be sufficiently strong to overcome the slope of the value function. Other than CPT won't be able to explain preferences for gambling and insurance. Therefore we cannot consider a simple functional as the polynomial of degree three proposed by Rieger and Wang. Actually if we process to a polynomial approximation at the order three, the slope at zero won't be strong enough to accommodate gambling and insurance behavior. A solution would consist in considering a polynomial characterized partly by small exponents because it permits to well captured the overweighting of small probabilities. In this work, we set the values of the exponents so that the curve is as close as possible to the original curve. Let the weighting function given by:

$$w(p) = ap + bp^{1.1} + cp^{1.15} + dp^{1.2} + ep^2 + fp^{2.5} + gp^6$$

with $a + b + c + d + e + f + g = 1$ such as $w(1) = 1$.

According to these elements we can rewrite $w$ as :

$$\begin{aligned} w(p) &= ap + bp^{1.1} + cp^{1.15} + dp^{1.2} + ep^2 + fp^{2.5} + (1 - a - b - c - d - e - f)p^6 \\ w(p) - p^6 &= a(p - p^6) + b(p^{1.1} - p^6) + c(p^{1.15} - p^6) + d(p^{1.2} - p^6) + e(p^2 - p^6) + j(p^{2.5} - p^6) \\ y &= ax_1 + bx_2 + cx_3 + dx_4 + ex_5 + fx_6 \end{aligned}$$

with $y = w(p) - p^6$, $x_1 = (p - p^6)$, ....,.$x_6 = (p^{2.5} - p^6)$

In order to estimate the coefficients $a$, ...., $f$, we built 1027 observations from the Tversky and Kahneman's weighting function[9]. Table 3 (respectively 4) displays the results of the estimates for gains (respectively losses)[10].

[9]We assume that $\gamma^+ = 0,61$ et $\gamma^- = 0,69$.

[10]To test the stability of the coefficients, we realized 14 regressions by removing randomly 5% of the observations. We mainly focused on $a$ which represents the slope of the function at zero. For gains, its average estimate is 2215.45 and the standard error equals to 1.70%.

In this study, we kept the estimate the closest to the mean.

Table 3: Results of the estimates for gains

| Variable | Coefficient | T-Stat | Prob |
|---|---|---|---|
| $x_1$ | 2 215.004 | 14.98 | 0.000 |
| $x_2$ | -19 080.29 | -15.23 | 0.000 |
| $x_3$ | 30 702.47 | 15.39 | 0.000 |
| $x_4$ | -13 963.80 | -15.55 | 0.000 |
| $x_5$ | 202.1941 | 18.84 | 0.000 |
| $x_6$ | -76.95053 | -20.95 | 0.000 |

Table 4: Results of the estimates for losses

| Variable | Coefficient | T-Stat | Prob |
|---|---|---|---|
| $x_1$ | 1295.432 | 15.57 | 0.000 |
| $x_2$ | -11 162.53 | -15.73 | 0.000 |
| $x_3$ | 17 972.97 | 15.91 | 0.000 |
| $x_4$ | -8 180.233 | -16.09 | 0.000 |
| $x_5$ | 119.3246 | 19.67 | 0.000 |
| $x_6$ | -45.40450 | -21.9 | 0.000 |

The weighting function is thus given by:

$$\begin{aligned} w(p) &= 2215,003p - 19080,29p^{1,1} + 30702,47p^{1,15} - 13963,8p^{1,2} \\ &\quad +202,1941p^2 - 76,95053p^{2,5} + 2,37243p^6 \end{aligned}$$

This functional form has a finite slope at zero and one. It is strictly increasing on [0,1] and satisfies the overweighting of small probabilities and underweighting of moderate and high probabilities.

## 5.2 A non-polynomial form

We underline previously that the limitations of the weighting function proposed by Tversky and Kahneman only concerns its slope at 0 and 1. Concerning the remainder of the interval, this specification perfectly characterizes attitudes towards probabilities. We can thus keep the global framework of these functional form and modify it lightly in a such way that the slopes at the extremities of the interval are not anymore infinite.
Let this functional form be given by:

$$w_1(p) = \frac{(p+a)^{\gamma_1}}{[(p+a)^{\gamma_1} + (1-(p+a))^{\gamma_1}]^{\frac{1}{\gamma_1}}} - b$$

with $a = 0 + \varepsilon$ , $w^{T+K}(a) = b$ and $0,3 < \gamma_1 < 1$.

This specification is stepest near zero and shallower in the middle. It satisfies the properties of subadditivity and is first concave then convex. The condition $w_1(0) = 0$ is satisfied. At last, its slope is finite at zero. That one can be written as:

$$a^{\gamma_1}((1-a)^{\gamma_1})^{\frac{-1}{\gamma_1}} \times \gamma_1 \left(\frac{1}{a} + \frac{(1-a)^{-1+\gamma_1} - a^{-1+\gamma_1}}{((1-a)^{\gamma_1} + a^{+\gamma_1})\gamma_1)}\right)$$

Note that on $[1-a,\, 1]$, $w_1(p)$ is not defined. We have to modifiy the specification for high values of $p$. Let's consider this functional form:

$$w_2(p) = \frac{(p-c)^{\gamma_2}}{[(p-c)^{\gamma_2} + (1-(p-c))^{\gamma_2}]^{\frac{1}{\gamma_2}}} + d$$

with $c = 0 + \varepsilon$ , $d$ such as $w_2(1) = 1$ and $w_2$ strictly increasing and convex.

The weighting function can thus be given by:

$$\begin{aligned} w(p) &= \frac{(p+a)^{\gamma_1}}{[(p+a)^{\gamma_1} + (1-(p+a))^{\gamma_1}]^{\frac{1}{\gamma_1}}} - b \quad \text{pour} \quad p \in [0,h] \\ &= \frac{(p-c)^{\gamma_2}}{[(p-c)^{\gamma_2} + (1-(p-c))^{\gamma_2}]^{\frac{1}{\gamma_2}}} + d \quad \text{pour} \quad p \in ]h,1] \end{aligned}$$

The value of $h$ is chosen in such a way that $w$ is strictly increasing on [0, 1]. $h$ is thus the solution of: $w_1(p) = w_2(p)$. $w$ is first strictly concave and then strictly convex only if $\gamma_1 > \gamma_2$[11]. At last, as the slope at 1 and 0 is identical in the specification proposed by Tversky and Kahneman, we assume that $w_1'(0) = w_2'(1)$.

In this study we set the values of $a$ and $c$ in such a way that the slope at 0 and 1 equals to 2 200 for gains and 1 300 for losses. These values are not arbitrarily chosen but correspond to the slopes we have obtained in the previous section. Thus for gains, we determine $a$ and $c$ by resolving the following system:

[11] If $\gamma_1 = \gamma_2$, $w$ re-becomes concave when $w_2$ replace $w_1$, whatever the value of $h$.

$$a^{\gamma_1}((1-a)^{\gamma_1})^{\frac{-1}{\gamma_1}} \times \gamma_1 \left(\frac{1}{a} + \frac{(1-a)^{-1+\gamma_1} - a^{-1+\gamma_1}}{((1-a)^{\gamma_1} + a^{+\gamma_1})\gamma_1)}\right) = 2200$$

$$\frac{((1-c)^{\gamma_2} + c^{\gamma_2})^{-\frac{1+\gamma_2}{\gamma_2}} \times \left((1-c)^{\gamma_2} \times c - c^{\gamma_2} + c^{1+\gamma_2}\right)}{(c-1) \times c} = 2200$$

Applying the same method for losses, we obtain the following weighting function:

$$w_1^+(p) = \frac{(p + 7.57 \times 10^{-10})^{0.61}}{[(p + 7.57 \times 10^{-10})^{0.61} + (1 - (p + 7.57 \times 10^{-10}))^{0.61}]^{\frac{1}{0.61}}} - 2.7307 \times 10^{-6}$$

for $p \in [0; 0.9999999]$

$$w_2^+(p) = \frac{(p - 5.585 \times 10^{-9})^{0.6}}{[(p - 5.585 \times 10^{-9})^{0.6} + (1 - (p - 5.585 \times 10^{-9}))^{0.6}]^{\frac{1}{0.6}}} + 1.8621 \times 10^{-5}$$

for $p \in ]0.9999999; 1]$

$$w_1^-(p) = \frac{(p + 2.721 \times 10^{-11})^{0.69}}{[(p + 2.721 \times 10^{-11})^{0.69} + (1 - (p + 2.721 \times 10^{-11}))^{0.69}]^{\frac{1}{0.69}}} - 5.1281 \times 10^{-8}$$

for $p \in [0; 0.9999999987]$

$$w_2^-(p) = \frac{(p - 1.8549 \times 10^{-10})^{0.68}}{[(p - 1.8549 \times 10^{-10})^{0.68} + (1 - (p - 1.8549 \times 10^{-10}))^{0.68}]^{\frac{1}{0.68}}} + 3.547 \times 10^{-7}$$

pour $p \in ]0.9999999987; 1]$

The specification of these two weighting functions has a finite slope at zero. It permits to avoid infinite subjective utility and thus overcomes the difficulties linked to the use of rank dependant models. These functional forms do not imply, as the one proposed by Rieger and Wang, no risk seeking behavior over unlikely gains and no risk aversion over unlikely losses. The slope at zero is sufficiently strong: the probability weighting can then over ride the curvature of the value function. CPT does not fail to explain gambling behavior[12].

## 6 Conclusion

The aim of this study was to determine how we can solve the St Petersburg Paradox in rank dependant models. First, we established that the solutions proposed in the literature lead to other kind of difficulties. We underlined that if we take them into account, the probability weighting won't be strong enough to compensate the concavity of the value function. In that case, CPT cannot accomodate both gambling and insurance behavior. This theory will then loose a major part of its descriptive power. In a second part, we proposed an alternative way to fix the infinite subjective utility's problem. As Rieger and Wang (2006), we suggested to consider an alternative weighting function whose slope at zero is not infinite. In that case, the subjective value of any prospects won't be infinitely high. Nevertheless, in order to preserve the fourforld pattern of risk attitudes, we set a specification whose shape dominates (for low probabilities) the value function. Thanks to this requirement, the overweighting of small probabilities reverses risk averse (respectively risk seeking) behavior for gains (respectively losses) generated by the value function. CPT won't fail to to provide a good description of individual behavior under risk.

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[12] In appendice, we show that the subjective utility of the *euromillions* lottery is positive for individuals whose preferences are represented by the weighting functions.

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# Appendice

Table 5 displays the results of *euromillions*' valuation by CPT with Camerer and Ho (1994) and Wu and Gonzales (1996) estimates[13].

[13] We consider a reference point of euro 2.

Table 5: Euromillions Valuation

| Probabilities | Gains | $v(x_i)$ Camerer Ho | $\pi_i$ Camerer Ho | $v(x_i)$ Wu Gonzales | $\pi_i$ Wu Gonzales |
|---|---|---|---|---|---|
| $1.311\times10^{-8}$ | 15 200 000 | 454.236 | $3.85332\times10^{-5}$ | 5426.9851 | $2.5322\times10^{-6}$ |
| $1.835\times10^{-7}$ | 422 857.14 | 120.695 | 0.000136992 | 842.59648 | $1.478610^{-5}$ |
| $2.753\times10^{-7}$ | 76 363.63 | 64.07134 | 0.000111006 | 346.01515 | $1.4926\times10^{-5}$ |
| $2.949\times10^{-6}$ | 5 128.2 | 23.58438 | 0.000581451 | 84.936411 | $9.935\times10^{-5}$ |
| $4.129\times10^{-5}$ | 242.27 | 7.60087 | 0.002775051 | 17.296997 | 0.0006838 |
| $6.194\times10^{-5}$ | 113.03 | 5.7125 | 0.002260728 | 11.578418 | 0.0006947 |
| 0.0001297 | 77.05 | 4.94189 | 0.003262787 | 9.4449954 | 0.0011434 |
| 0.0018181 | 28.05 | 3.340792 | 0.020489824 | 5.4477971 | 0.0095197 |
| 0.0018587 | 23.67 | 3.120927 | 0.01197572 | 4.9507266 | 0.0069054 |
| 0.0027247 | 17.25 | 2.740316 | 0.01302709 | 4.1237036 | 0.0084206 |
| 0.0098039 | 10.30 | 2.188255 | 0.0311254 | 3.0059076 | 0.0235416 |
| 0.0263157 | 9.12 | 2.067367 | 0.04819050 | 2.7751710 | 0.0451533 |
| 0.9572 | 0 | -2.90779 | 0.7638842 | -3.226399 | 0.8742871 |
| **V(X)** | | | **-1.80** | | **-2.43** |

In both case the subjective utility of one *Euromillions* ticket is negative. We can generalize this result and show (for any value of $\gamma$ greater than 0.4) that if we consider a power coefficient for the value function greater than the power coefficient of the weighting function, the subjective utility of this game is always negative.

Table 6 displays the results of *euromillions*' valuation by considering the weighting functions defined in section 5[14].

Table 6: Euromillions Valuation

| Probabilities | Gains | $v(x_i)$ | $\pi_i^1$ | $\pi_i^2$ |
|---|---|---|---|---|
| $1.311\times10^{-8}$ | 15 200 000 | 2 089 403.44 | $9.90096\times10^{-6}$ | $1.3371\times10^{-6}$ |
| $1.835\times10^{-7}$ | 422 857.14 | 89 340.55 | $9.50558\times10^{-5}$ | $6.5217\times10^{-5}$ |
| $2.753\times10^{-7}$ | 76 363.63 | 19 012.08 | 0.00011415 | $5.7184\times10^{-5}$ |
| $2.949\times10^{-6}$ | 5 128.2 | 1839.16 | 0.000864002 | 0.0003245 |
| $4.129\times10^{-5}$ | 242.27 | 124.46 | 0.005956235 | 0.0017513 |
| $6.194\times10^{-5}$ | 113.03 | 63.09 | 0.005239292 | 0.0015393 |
| 0.0001297 | 77.05 | 44.7 | 0.007202387 | 0.0023238 |
| 0.0018181 | 28.05 | 17.61 | 0.029688009 | 0.0160782 |
| 0.0018587 | 23.67 | 14.98 | 0.00833742 | 0.0101514 |
| 0.0027247 | 17.25 | 10.99 | 0.00547187 | 0.0115014 |
| 0.0098039 | 10.30 | 6.44 | 0.01083462 | 0.0290591 |
| 0.0263157 | 9.12 | 5.62 | 0.0373534 | 0.0485568 |
| 0.9572 | 0 | -4.14 | 0.8862328 | 0.8633637 |
| **V(X)** | | | **31.74** | **33.36** |

[14]We consider a reference point of euro 2.

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