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# CAPITAL PROTECTED NOTES FOR LOSS AVERSE INVESTORS: A COUNTERINTUITIVE RESULT

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#### Abstract

Capital protected notes are very popular structured products since the internet bubble burst in 2000. Investors are protected against large losses they could suffer if they were investing directly in the underlying index or portfolio of stocks. It then seems intuitive that such products are attractive for loss averse investors. However, using a simple version of cumulative prospect theory, we show that these products are not attractive when the investor takes either the underlying index or the risk-free investment as the reference point. She always prefer an investment in the index or in the risk-free portfolio, depending on her coefficient of loss aversion.

#### JEL classification: G11

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#### **1 INTRODUCTION**

In recent years, especially after the internet bubble burst in 2000, a lot of structured products have been issued by banks and other financial institutions. They are usually built as portfolios of more basic financial securities, like stocks, indices or options on diverse underlying assets. Their purpose is to cater to the needs of different categories of customers. Retail customers are then offered sophisticated positions in options without having to build complex portfolios by themselves. A large proportion of these products offer partial or total capital protection, at least in nominal terms.

Many of these products are quoted on the AMEX or the NYSE. To take a simple example among others, Morgan Stanley issued in July 2003 a capital protected note indexed on the S&P  $500^1$ . It matures in January 2011, insures 100 % of the initial capital at the maturity date, pays no interest, but provides 80 % of the index return (called the participation rate) at the maturity date if the performance<sup>2</sup> of the index is positive. It is then equivalent to a direct investment in the index, protected by an at-the-money put option. The put option is financed by selling 20% (called the concession rate) of an at-the-money call on the index. In general, issuers of such notes promote the capital protection by comparing their products to a direct investment in the index, especially in case of a large drop. In others words, they take as the reference investment the index or, more generally, the underlying portfolio on which the return of the note is indexed.

If investors are assumed loss averse, it seems very natural for banks, on a com-

<sup>&</sup>lt;sup>1</sup>The prospectus can be found on www.amex.com.

<sup>&</sup>lt;sup>2</sup>Here, the performance doesn't include dividends.

mercial point of view, to propose capital protected notes which prevent investors to suffer large losses. However, the key question is to know what is the reference to which the notes are compared. In this paper, we then consider investors obeying a simple version of cumulative prospect theory<sup>3</sup> (CPT in the following) and compare the utility they get from investing in the notes with the one obtained by investing in the index or in the risk-free asset. As value comes from gain and losses in CPT, the choice of the reference point is important to evaluate a risky prospect. Nevertheless, the reference point chosen by an investor is not independent of the way these products are presented and advertised by issuers. For example, insisting on the guarantee provided by the contract induces a comparison with a direct investment in the underlying index. On the contrary, if the focus is put on the participation rate and then on the possibility to obtain high returns, it assumes an implicit comparison with the status quo or with an investment in the risk-free asset.

In this paper, we first show that if the investor takes the underlying index as the reference point, it is never optimal to invest in such notes. The reason is very simple. When the index is taken as the reference, the results provided by the note are perceived as gains when the index value falls below the guaranteed level, that is to say, when the insurance provided by the issuer starts to work. But when the index level increases, the investor loses a part of the performance corresponding to the concession rate. In other words it is perceived as a loss.

If the reference point of the investor is the status quo (or an investment in the riskfree asset), we find that the valuation function is maximized for an investment in the

<sup>&</sup>lt;sup>3</sup>Kahneman and Tversky (1979) and Tversky and Kahneman (1992).

index or in the risk-free investment, depending on the level of loss aversion. It means that these structured products never seem attractive to the loss averse investor.

Most studies on structured products concern the question of pricing. Wilkens *et al.* (2003) and Stoimenov and Wilkens (2005) show that structured products are often overpriced on the German market, especially on the primary market. The overpricing tends to decrease on the secondary market as the maturity date approaches. Comparable results are obtained by Burth *et al.* (2001) and Grünbichler and Wohlend (2005) on the Swiss market.

In a theoretical framework, Carlin (2007) predicts that this increase in profits for the issuers is partly due to the complexity of structured products. However, Horst and Veld (2008) detect a clear overpricing when comparing simple warrants issued by banks to the corresponding exchange traded options. Saying that structured products are overpriced is essentially based on a no arbitrage argument. A structured product is, in most cases, a portfolio of an underlying asset and options on this asset. Consequently, option pricing models and associated implicit volatilities can be used to value these products and conclude if they are overpriced or not. However, usual pricing models don't take into account the fact that banks provide a service to customers in building and managing these sophisticated portfolios. In other words, overpricing can simply be a payment for the service provided by the issuer.

There are in fact many ways to design structured products but we will focus on one of the most simple categories. We concentrate our attention on capital protected notes which guarantee a given proportion of the initial capital and retain a given part of the potential profits. Obviously, issuers and customers have conflicting interests in the design of the notes. The former want to minimize the return paid and the risk borne when the latter looks for an attractive risk/return couple, that is to say, an attractive expected return and a high level of capital protection. The two steps in building structured products are the following (Breuer and Perst, 2007):

1) Evaluation of the costs of designing specific risk/return profiles by means of theoretic tools based on arbitrage considerations;

2) Evaluation of the utility gains for the customer.

The way utility is measured is essential at this stage and is closely linked to investors' preferences. For example, the chance of incurring a large loss, relative to what is expected, was ranked as the first attribute of risk in a survey by Olsen (1997). Using interview data from a sample of traders and managers in four investment banks, Willman *et al.* (2002) also conclude that managers focus on avoiding losses rather than making gains

Searching capital protection can also be justified, at the theoretical level, by loss aversion. It is a fundamental characteristic of individual agents in cumulative prospect theory (CPT in the following) developed by Kahneman and Tversky (1979) and Tversky and Kahneman (1992). This model of decision-making under risk is one of the most promising alternative approaches to the standard expected utility model because it proposes reasonable solutions to many financial puzzles. It was essentially motivated by the numerous observations of violations of the von Neumann and Morgenstern (1947) axioms, but also by the observation that many agents acquire simultaneously insurance contracts and tickets for unfair lotteries.

In the present paper, we analyze the optimal structure of the capital protected notes described above, for a client obeying CPT, under some constraints on the issuer side, the simplest one being that the initial amount paid by the investor is the "fair" date-0 value of the final payoff. The structured products considered here are characterized by two parameters: the first one is the proportion of the initial investment guaranteed to the investor at the maturity date, the second one is the participation rate in case of an increase of the underlying index. Even if two sets of parameters are equivalent in strictly financial terms (using no arbitrage arguments), an investor obeying CPT will prefer one of the two sets, taking into account her objective function. Therefore, our contributions are the following: first, we solve the optimization problem of a CPT investor for different reference points (the underlying index and the risk-free investment). We show that the underlying index is never a "good" reference to promote such notes because they are always dominated by a direct investment in the underlying index. Third, when the reference of the investor is the risk-free investment, we show that she prefers either to invest in the index or in the risk-free asset, depending on her coefficient of loss aversion.

The paper is organized as follows. Section 2 describes the final payoffs and the arbitrage-free price of the capital protected notes under consideration. Section 3 briefly summarizes the main features of CPT and justifies our assumptions concerning the parameters of the valuation function. Section 4 solves the optimal design problem for CPT investors with different reference points and presents our essential results. Finally, section 5 concludes the paper and proposes some extensions.

#### 2 CAPITAL PROTECTED NOTES

We consider a capital protected note (henceforth CPN) linked to an index (or a portfolio of stocks). It provides some capital protection to the investor but this guarantee is financed by giving up a given proportion of the profits, called the concession rate.

### 2.1 FINAL PAYOFF AND ARBITRAGE-FREE PRICE OF CPNs

We assume that the underlying index value S is driven by a geometric brownian motion  $S = (S_t, t \in [0; T])$  with parameters  $(\mu, \sigma)$ , where T is the maturity date of the CPN X under consideration. The stochastic process of the value of X is also denoted  $X = (X_t, t \in [0; T]), X_T$  being the final payoff received by the investor at date T.

 $X_T$  is defined by:

$$X_T = \theta S_0 + \max(S_T - \theta S_0; 0) - (1 - \alpha) \max(S_T - \max(\theta; 1)S_0; 0)$$
(1)

where  $\theta S_0$  is the guaranteed amount at the maturity date and  $\alpha$  is the participation rate. The strike price  $\max(\theta; 1)S_0$  appears in the last term of the RHS of equation (1) to take into account the case  $\theta > 1$ .

The final value of the index is given by:

$$S_T = S_0 \exp\left[\left(\mu - \frac{\sigma^2}{2}\right)T + \sigma Z_T\right]$$

where  $Z = (Z_t, t \ge 0)$  denotes a standard brownian motion.

It is assumed that  $S_0$  is the initial amount paid by the investor to buy the note, and also the initial value of the underlying index. The strategy is then characterized by the two parameters  $(\alpha, \theta)$ . In fact, if  $\theta \leq 1$  and the final price  $S_T$  is greater than  $S_0$ , the investor receives  $S_0 + \alpha(S_T - S_0)$ . If  $S_T$  lies between  $\theta S_0$  and  $S_0$ , the investor is paid  $S_T$ . Finally, in the case  $\theta > 1$ , the investor receives  $\theta S_0 + \alpha(S_T - \theta S_0)$  when the underlying final price is above  $\theta S_0$ . It is worth to notice that in most practical cases, the guaranteed amount is at most  $S_0$  or, in other words, usual contracts correspond to  $\theta \leq 1$ . Figure 1 shows the final payoff of the note as a function of the underlying value when  $S_0 = 1, \theta = 0.9$  and  $\alpha = 0.6$ .

#### Figure 1 around here

Under the no arbitrage assumption we get:

$$X_0 = e^{-rT} E_Q \left[ X_T \right] \tag{2}$$

where r is the continuous risk-free rate and  $E_Q[.]$  is the expectation operator with respect to the risk-neutral probability Q.

Equation (2) is equivalent to:

$$X_0 = e^{-rT}\theta S_0 + C(\theta S_0, r) - (1 - \alpha)C(\max(\theta, 1)S_0, r)$$
(3)

where C(Y, r) is the date-0 Black-Scholes price of a call option with maturity T, a strike price equal to Y and an interest rate equal to r.

In a perfect market,  $X_0 = S_0$  since  $S_0$  is the amount initially paid by the investor to buy the structured product. It implies that the two parameters  $(\alpha, \theta)$  are linked by the relationship:

$$S_0 \left( 1 - \theta e^{-rT} \right) = C(\theta S_0, r) - (1 - \alpha)C(\xi S_0, r)$$
(4)

with  $\xi = \max(\theta; 1)$ .

We then have the following preliminary lemma.

**Lemma 1** The parameters  $(\alpha, \theta)$  satisfying equation (4) do not depend on  $S_0$ .

**Proof.** Obvious, because the call prices are homogeneous of degree 1 in  $S_0$ . Lemma 1 allows to normalize the problem with  $S_0 = 1$ . This assumption will be used in the rest of the paper. Equation (4) is then equivalent to:

$$1 - \theta e^{-rT} = C(\theta, r) - (1 - \alpha)C(\xi, r)$$
(5)

# 2.2 RELATIONSHIP BETWEEN CAPITAL PROTECTION AND PARTICIPATION RATE

The Black-Scholes model allows to write equation (5) as follows:

$$1 - \theta e^{-rT} = N(d_1(\theta, r)) - \theta e^{-rT} N(d_2(\theta, r)) - (1 - \alpha) (N(d_1(\xi, r)) - \xi e^{-rT} N(d_2(\xi, r)))$$
(6)

where, for  $\gamma = \theta$  or  $\xi$ :

$$d_1(\gamma, r) = \frac{-\ln(\gamma) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \text{ and } d_2(\gamma, r) = d_1(\gamma, r) - \sigma\sqrt{T}$$

We specify the two variables  $(\theta, r)$  or  $(\xi, r)$  in the expression of  $d_1$  and  $d_2$  because we will use, in section 4, other formulas with  $(\theta, \mu)$  or  $(\xi, \mu)$ .

In equation (6),  $\alpha$  can be expressed in an explicit way as a function of  $\theta$ . We get:

$$\alpha = 1 - \frac{N(d_1(\theta, r)) - \theta e^{-rT} N(d_2(\theta, r)) - (1 - \theta e^{-rT})}{N(d_1(\xi, r)) - \xi e^{-rT} N(d_2(\xi, r))}$$
(7)

$$= 1 - \frac{\theta e^{-rT} N(-d_2(\theta, r)) - N(-d_1(\theta, r))}{N(d_1(\xi, r)) - \xi e^{-rT} N(d_2(\xi, r))}$$
(8)

Equation (8) comes from the put-call parity relationship. The concession rate  $1 - \alpha$  is then the ratio of two option prices, a put with a strike price  $\theta$  and a call with a strike price  $\xi$ .

It is worth to note that if  $\theta = e^{rT}$ , the no-arbitrage assumption implies  $\alpha = 0$ . In fact, the put price and the call price are equal when the strike price is  $e^{rT}$ . The CPN becomes a risk-free asset in this case. If  $\theta = 0$  (no guarantee is provided), then  $\alpha = 1$ . If it was not the case, there would be an arbitrage opportunity.

Finally, when  $\theta \in \left]1; e^{rT}\right[$ ,  $\alpha$  is simplified in:

$$\alpha = \frac{(1 - \theta e^{-rT})}{N(d_1(\theta, r)) - \theta e^{-rT}N(d_2(\theta, r))}$$
(9)

This case has not much practical interest since almost all products issued in recent years guarantee 100 % of the initial investment or less. It is essentially linked to the

"low" interest rates and to the maturity of the note which is often short or medium. However, it is easy to imagine a long-term issue in a high-interest rate period where it is necessary for the issuer to propose more than 100 % of the initial investment to attract investors. An other more common way to provide more than a 100 % protection is to pay yearly coupons on the note and to keep  $\theta = 100$  %. This case will not be analyzed here.

The sensitivity of  $\alpha$  with respect to the level of capital protection is given in proposition 3. It is based on the following lemma which recalls, with our notations and assumptions, the derivatives of option prices with respect to the exercise price.

**Lemma 2** Let  $\sigma$  and T be positive constants, N(x) denote the cumulative distribution function of a standard gaussian variable and C and P defined as:

$$C(a,b) = N(d_1(a,b)) - ae^{-bT}N(d_2(a,b))$$
$$P(a,b) = ae^{-bT}N(-d_2(a,b)) - N(-d_1(a,b))$$

with

$$d_1(a,b) = \frac{-\ln(a) + \left(b + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$
 and  $d_2(a,b) = d_1(a,b) - \sigma\sqrt{T}$ 

The derivatives of C and P with respect to a are given by:

$$\frac{\partial C}{\partial a} = -e^{-bT}N(d_2(a,b)) \text{ and } \frac{\partial P}{\partial a} = e^{-bT}N(-d_2(a,b))$$

**Proof.** See the Appendix.

Notations of lemma 2 lead to write the concession rate as the ratio of two option prices in the following form:

$$1 - \alpha = \frac{P(\theta, r)}{C(\xi, r)} \tag{10}$$

The derivative of  $\alpha$  with respect to  $\theta$  can now be easily calculated.

#### **Proposition 3**

$$\frac{\partial \alpha}{\partial \theta} = -\frac{e^{-rT}N(-d_2(\theta, r))}{C(1, r)} < 0 \quad if \ \theta < 1$$

$$= e^{-rT}\frac{N(d_2(\theta, r)) - N(d_1(\theta, r))}{C(\theta, r)^2} < 0 \quad if \ \theta \in \left[1; e^{rT}\right]$$
(11)

#### **Proof.** See the appendix $\blacksquare$

The results of proposition 3 are very intuitive because when the level of capital protection increases, the cost of the insurance provided to the investor also increases, and so does the concession rate. Consequently, the participation rate decreases when the proportion of capital guaranteed to the investor increases. Figure 2 shows the evolution of  $\alpha$  as a function of  $\theta$  for the following parameters: r = 4%, T = 1,  $\sigma = 20\%$  and  $\theta \in [0.5; e^{rT}]$ . The function  $\alpha(\theta)$  is not differentiable at  $\theta = 1$  as can be observed in proposition 3. All the points on the curve are equivalent in strictly financial terms as they correspond to a structured product whose value is equal to the initial investment. The question we will address later on concerns the choice of a loss averse investor if he is offered such contracts. Which contract is preferred among the set of "fair" contracts?

#### Figure 2 around here

We also observe on the figure that for  $\theta < 0.6$ ,  $\alpha$  is almost 1. It is simply explained by the fact that such an exercise price corresponds to a widely out of the money put. The probability that  $S_T < \theta$  is small (the maturity being one year in this example) and the investor is not ready to pay much for such a protection. On the other side, a complete protection of the initial amount (in nominal terms) leads to a concession rate of about 60%. Due to the short maturity of the note, such a protection is "expensive". When the risk-free rate is positive, increasing the horizon decreases the probability of a negative return and then increases the corresponding  $\alpha$ .

#### **3 CUMULATIVE PROSPECT THEORY**

# 3.1 THE VALUATION FUNCTION OF PROSPECT THE-ORY

CPT was developed by Kahneman and Tversky (1979) and Tversky and Kahneman (1992) as an alternative theory of decision making under risk, to take into account the many violations of the usual assumptions of expected utility theory. The four essential features of CPT are the following:

1) Investors value risky prospects with respect to a reference point which may be the investor's current wealth, for example when the outcomes are obtained immediately, or the current wealth capitalized at the risk-free rate when outcomes are obtained at a future date, as in Barberis *et al.* (2001). However, other reference points may be used by investors, depending on the set of alternative available prospects.

2) Losses and gains are weighted differently in the valuation function. The per-

ceived disutility of a 100\$ loss is greater than the utility of 100\$ gain.

3) Agents are risk-averse in the domain of gains and risk-lovers in the domain of losses. In other words, when a portfolio generates losses, agents are ready to gamble to avoid the largest ones.

4) Investors tend to overweigh the small probabilities of extreme events.

In this paper we are essentially interested in the first two points for the following reasons:

- we want to focus on the role of loss aversion in the attractiveness of capital protected notes;

- we want to compare different reference points in the evaluation of the notes for loss averse investors.

The valuation function v proposed by Tversky and Kahneman (1992) is defined by:

$$v(x) = \begin{cases} x^{\beta} \text{ if } x \ge 0\\ -\lambda (-x)^{\beta} \text{ if } x < 0 \end{cases}$$
(12)

where  $\lambda > 1$  is the loss aversion coefficient and  $\beta < 1$  is aimed at taking into account the diminishing sensitivity to gains and losses. In equation (12), x denotes a gain or a loss with respect to the reference point which is discussed hereafter. Tversky and Kahneman (1992) obtained  $\lambda = 2.25$  and  $\beta = 0.88$  using experimental data. It means that v is only slightly concave over gains and slightly convex over losses. To simplify the formulation of the problem and to obtain analytical results, we will choose  $\beta = 1$ . Under this assumption, v is a piecewise linear valuation function.

## 3.2 ASSUMPTIONS ON THE REFERENCE POINT AND THE VALUATION FUNCTION

In the present paper, our main goal is to analyze if CPNs improve the utility of investors obeying cumulative prospect theory. Issuers of CPNs often promote capital protection by comparing the payoffs to those of a direct investment in the underlying asset or by emphasizing the upside potential by means of a comparison with a riskfree investment. In the first case, they insist on the guaranteed capital and in the second case, they compare to an investment in a risk-free asset.

The question is to know if these reference points are good choices to promote CPNs for loss averse investors. We will show that it is not the case. When investors obey our simple version of prospect theory, they always prefer a direct investment in the index or an investment in the risk-free asset, depending on their coefficient of loss aversion.

Let  $X^*$  denote the reference level of final wealth. According to CPT when  $\beta = 1$ , investors are supposed to maximize:

$$E_P[v(X_T - X^*)] = E_P[(X_T - X^*)_+] - \lambda E_P[(X^* - X_T)_+]$$
(13)

As mentioned before, there are several candidates for  $X^*$ . The first one is the terminal index level  $S_T$ . Focusing on capital protection induces a direct comparison between the probability distribution of the final value of the CPN and the corresponding distribution for the final index value.

Other natural reference points are the initial wealth capitalized at the risk-free

rate or simply the status quo, which is often referred to by issuers, especially when the design of the CPN corresponds to  $\theta = 1$ . In this case, advertising is focused on the "no loss" investment. Whatever happens to the index, you will get back your initial investment!

In the following section, we examine these possibilities but we merge the two last ones in subsection 4.2 by assuming a zero risk-free rate.

# 4 THE OPTIMAL CHOICE OF A LOSS AVERSE INVESTOR

We address now the question of the optimal contract for a loss averse investor obeying our simple version of CPT. The main result is that investing in a CPN, as designed before, is never optimal if either the final value of the index or the risk-free investment are taken as the reference portfolio. This result does not seem very intuitive. When the index value is chosen as the reference it appears because, in a sense to be precised later on, gains and losses are reversed. With the other reference, the coefficient of loss aversion determines the optimal investment which is either the index or the risk-free asset.

#### 4.1 THE INDEX AS THE REFERENCE POINT

When  $X^* = S_T$ , the expected valuation function is written:

$$E_P[v(X_T - S_T)] = E_P[(X_T - S_T)_+] - \lambda E_P[(S_T - X_T)_+]$$

After normalizing the initial investment  $S_0$  to 1, we know that:

$$X_T = \theta + \max(S_T - \theta; 0) - (1 - \alpha) \max(S_T - \xi; 0)$$

Therefore

$$X_T - S_T = \theta - S_T + \max(S_T - \theta; 0) - (1 - \alpha) \max(S_T - \xi; 0)$$
(14)  
$$= \begin{cases} -(1 - \alpha)(S_T - \xi) \text{ if } S_T > \xi \\ 0 \text{ if } \theta \le S_T \le \xi \\ \theta - S_T \text{ is } S_T < \theta \end{cases}$$

This formulation is interesting due to the choice of the reference point. Gains appear if the price of the underlying falls below the guaranteed amount  $\theta$  and these gains correspond to the loss of the issuer which guarantees the given amount when she invests in the underlying asset.

From these equalities, we deduce:

$$E[v(X_T - S_T)] = E_P[(\theta - S_T)_+] - \lambda(1 - \alpha)E_P[(S_T - \xi)_+]$$
$$= e^{\mu T}[P(\theta, \mu) - \lambda(1 - \alpha)C(\xi, \mu)]$$

The second equality is obtained by using notations of lemma 2.

Maximizing the expected value function of the investor may now be written as a

one-variable problem by using the expression of the concession rate in equation (10).

$$\max_{\theta} F(\theta) = \max_{\theta} e^{\mu T} \left[ P(\theta, \mu) - \lambda \frac{P(\theta, r)}{C(\xi, r)} C(\xi, \mu) \right]$$
(15)

The relationship between  $\alpha$  and  $\theta$  is in fact the participation constraint of the issuer.

#### 4.1.1 THE CASE OF PARTIAL PROTECTION $\theta < 1$

When  $\theta < 1$ ,  $F(\theta)$  is simplified in:

$$F(\theta) = e^{\mu T} \left[ P(\theta, \mu) - \lambda \frac{C(1, \mu)}{C(1, r)} P(\theta, r) \right]$$
(16)

It allows to prove the following result

**Proposition 4** It is never optimal for a loss averse investor to buy the capital protected note as long as the risk premium on the index is positive. Moreover, for  $\theta < 1$ we have:

$$\frac{\partial F}{\partial \theta} = N(-d_2(\theta,\mu)) - \lambda e^{(\mu-r)T} \frac{C(1,\mu)}{C(1,r)} N(-d_2(\theta,r)) < 0$$
(17)

where f denotes the density of the standard gaussian distribution.

**Proof.** See the Appendix  $\blacksquare$ 

This result is not very intuitive because the disutility of the investor increases when the proportion of capital protected by the note increases. However, the reference being the index, an increasing level of protection generates a higher concession rate which is perceived as a loss when the index value increases. Moreover, on the side of perceived gains, that is when the index value decreases, the amount of gains is limited because the index value cannot fall below 0. On the side of losses (increase in the index value), there is no theoretical limit to perceived losses.

#### 4.1.2 THE CASE OF FULL PROTECTION

When  $\theta \ge 1$ , the final payment of the note is written as follows:

$$X_T = \theta + \alpha \max(S_T - \theta; 0)$$

Consequently, using the equality  $\xi = \theta$ , we obtain:

$$F(\theta) = e^{\mu T} \left[ P(\theta, \mu) - \lambda \frac{P(\theta, r)}{C(\theta, r)} C(\theta, \mu) \right]$$

As in the proof of proposition 4,  $r < \mu$  implies  $P(\theta, \mu) < P(\theta, r)$  and  $C(\theta, r) < C(\theta, \mu)$ . Therefore,  $F(\theta) < 0$  for  $\theta > 0$  as far as  $\lambda > 1$ . As F(0) = 0, investing in the note is not valuable to the loss averse investor.

#### 4.2 THE STATUS QUO AS THE REFERENCE POINT

We consider now the case of the status quo, chosen as the reference point in many experiments. In our framework, it can also be viewed as an investment in the risk-free asset when the interest rate is 0. In this section we then assume that r = 0 and  $\mu$  is simultaneously the expected return on the index and the risk premium. In this case we know that  $\theta$  cannot be greater than 1; consequently  $\xi = 1$ . Moreover, we have the following equivalence:

$$X_T - 1 > 0 \Leftrightarrow S_T > 1 \tag{18}$$

The gain/loss of the note with respect to the status quo is then:

$$X_T - 1 = \theta - 1 + \max(S_T - \theta; 0) - (1 - \alpha) \max(S_T - 1; 0)$$
(19)  
$$= \begin{cases} \alpha(S_T - 1) \text{ if } S_T > 1\\ S_T - 1 \text{ if } \theta \le S_T \le 1\\ \theta - 1 \text{ is } S_T < \theta \end{cases}$$

The interpretation is rather intuitive. The participation rate being  $\alpha$  and the reference point being  $S_0 = 1$ , the return on the index must be positive for the note to generate a positive return, as stated in relation (18).

The expected valuation function, denoted as  $G(\theta)$ , is now written as:

$$\max_{\theta} G(\theta) = E_P \left[ (X_T - 1)_+ \right] - \lambda E_P \left[ (1 - X_T)_+ \right]$$
(20)

The following technical lemma will be useful to calculate the expected valuation function of the investor.

**Lemma 5** If g denotes the density of  $S_T$ , p < q two constants and  $z \leq q$ , we get:

$$\int_{p}^{q} (z-x)g(x)dx = z \left[ N(-d_{2}(q,\mu)) - N(-d_{2}(p,\mu)) \right]$$

$$-e^{\mu T} \left[ N\left( -d_{1}(q,\mu) \right) - N(-d_{1}(p,\mu)) \right]$$
(21)

When z = q, this equality is written:

$$\int_{p}^{q} (q-x)g(x)dx = e^{\mu T}[P(q,\mu) - P(p,\mu)] - (q-p)N(-d_2(p,\mu))$$
(22)

**Proof.** See the appendix  $\blacksquare$ 

Relationships (19) and lemma 5 lead to:

$$G(\theta) = \alpha E_P \left[ (S_T - 1) \mathbf{1}_{]1;+\infty[} \right] - \lambda E_P \left[ (1 - S_T) \mathbf{1}_{\{S_T \in [\theta;1[\}]} \right]$$
(23)  
$$-\lambda (1 - \theta) P(\{S_T < \theta\})$$
  
$$= \alpha e^{\mu T} C(1,\mu) - \lambda \left[ e^{\mu T} [P(1,\mu) - P(\theta,\mu)] - (1 - \theta) N(-d_2(\theta,\mu)) \right]$$
$$-\lambda (1 - \theta) P(\{S_T < \theta\})$$

But we know that  $P(\{S_T < \theta\}) = N(-d_2(\theta, \mu))$ . The final expression of  $G(\theta)$  is then:

$$G(\theta) = e^{\mu T} \left[ \alpha C(1,\mu) - \lambda \left( P(1,\mu) - P(\theta,\mu) \right) \right]$$
(24)  
=  $e^{\mu T} \left[ \left( 1 - \frac{P(\theta,0)}{C(1,0)} \right) C(1,\mu) - \lambda \left( P(1,\mu) - P(\theta,\mu) \right) \right]$ 

The second equality comes from the definition of  $\alpha$  in equation (10).

Equation (24) implies that G(1) = 0 because P(1,0) = C(1,0). This result is not surprising because a complete guarantee  $\theta = 1$  implies a null participation rate when r = 0. Consequently the portfolio corresponding to  $\theta = 1$  is equal to the reference portfolio (the risk-free investment). When  $\theta = 0$ ,  $P(0, 0) = P(0, \mu) = 0$  and we deduce:

$$G(0) = e^{\mu T} \left[ C(1, \mu) - \lambda P(1, \mu) \right]$$

It appears that for  $\lambda > 1$  sufficiently high, this quantity is negative, leading to an optimal choice corresponding to a strictly positive  $\theta$ . It is economically sound that a high level of loss aversion leads to a positive level of guarantee. However, the question is to know if the optimal  $\theta$  lies in the open interval ]0; 1[. On the other side, for low levels of loss aversion, G(0) > 0 = G(1). The reference portfolio is not optimal but we have also to check if  $\theta = 0$  is an optimum or not. The following proposition answers the two questions.

**Proposition 6** The optimal value of  $\theta$  is either 0 or 1 depending on the value of  $\lambda$ .

**Proof.** See the appendix  $\blacksquare$ 

The derivative of G with respect to  $\theta$  is given by:

$$\frac{\partial G}{\partial \theta} = e^{\mu T} \left[ -\frac{C(1,\mu)}{C(1,0)} \frac{\partial P(\theta,0)}{\partial \theta} + \lambda \frac{\partial P(\theta,\mu)}{\partial \theta} \right]$$
$$= e^{\mu T} \left[ -\frac{C(1,\mu)}{C(1,0)} N(-d_2(\theta,0)) + \lambda e^{-\mu T} N(-d_2(\theta,\mu)) \right]$$

Therefore:

$$\frac{\partial G}{\partial \theta} = 0 \Leftrightarrow \frac{N(-d_2(\theta,\mu))}{N(-d_2(\theta,0))} = \frac{e^{\mu T}C(1,\mu)}{\lambda C(1,0)}$$
(25)

Figure 4 provides the ratio in the LHS of equality (25) as a function of  $\theta$ . The risk

premium is 6 %, the volatility 20 % and the maturity is 1 year. Obviously the ratio tends to 0 when  $\theta$  tends to 0 because  $\mu > 0$ . Moreover, the ratio is always below 1 for the same reason. Concerning the RHS of equation (25), we observe that  $C(1,\mu) > C(1,0)$  and  $\mu > 0$  so  $e^{\mu T}C(1,\mu)/C(1,0) > 1$ . Consequently,  $\partial G/\partial \theta$  may be equal to 0 only if the loss aversion coefficient is sufficiently large. Using the same parameters, the RHS of equation (25) is equal to 0.645. When  $\lambda = 2$  we get  $\theta \approx 85\%$ . However, as can be seen in the proof of proposition 6, the second derivative of G is positive at the stationary point and this point is then not a maximum but a minimum.

#### Figure 4 around here

Figure 5 illustrates this point with the same parameters as before. We observe that G decreases up to the stationary point and then becomes increasing, illustrating the convexity of G around the stationary point.

#### Figure 5 around here

#### 5 CONCLUDING REMARKS

In this paper, we obtained a rather counterintuitive result. We showed that loss averse investors never find optimal to invest in a capital protected note when their reference portfolio is either the underlying asset or the risk-free investment. They prefer to directly invest in one of the two reference portfolios, depending on their coefficient of loss aversion. Nevertheless, these products are very popular and this questions the assumptions of the usual theoretical models. The first one is the geometric brownian assumption for the index value, a standard one in the world of options since it allows to use the Black-Scholes model. Using fat tailed distributions of returns could possibly explain the popularity of CPNs for loss averse investors but, at the same time, it is well known that financial institutions commonly use the Black-Scholes model to price products with optional features. The second assumption that may be questioned is the valuation function used in the paper. Using a piecewise linear function may induce corner solutions, as it is the case in standard portfolio models. However, it is worth to note that in our paper the valuation of the CPN is **not linear** in  $\theta$ ; consequently, the usual drawback of piecewise linear utility functions cannot be used to cast doubt on our results. Moreover, many simulations (not included in the paper) with different values of  $\beta$  (the power in the valuation function of Tversky and Kahneman (1992)) lead to the same "all or nothing" solution.

It suggests the analysis of other structured products like the accelerating returns notes or the BULS<sup>4</sup> (Bullish Underlying Linked Securities) to get more insights on the optimal design of structured notes for loss averse investors. At a more general level, the question is to find, for any given reference, the dominating portfolios. As the payoff of any index linked portfolio using plain-vanilla options can be represented by a picewise linear function, the question to find such functions that dominate a given reference portfolio in the sense of CPT.

#### Appendix

#### Proof of lemma 2

<sup>&</sup>lt;sup>4</sup>See for example Chen and Wu (2007).

We give up the arguments (a, b) to simplify the notations. Let f(x) denote the density of the standard gaussian distribution:

$$\frac{\partial C}{\partial a} = \frac{\partial d_1}{\partial a} f(d_1) - a e^{-bT} \frac{\partial d_2}{\partial a} f(d_2) - e^{-bT} N(d_2)$$

As  $\frac{\partial d_1}{\partial a} = \frac{\partial d_2}{\partial a}$  it is sufficient to prove that  $f(d_1) - ae^{-bT}f(d_2) = 0$ .

$$f(d_1) = f(d_2 + \sigma\sqrt{T}) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(d_2 + \sigma\sqrt{T}\right)^2\right]$$
$$= f(d_2) \exp\left(-\frac{\sigma^2 T}{2}\right) \exp\left(\ln(a) - \left(b - \frac{\sigma^2}{2}\right)T\right) = ae^{-bT}f(d_2)$$

The arguments are the same for P.

$$\frac{\partial P}{\partial a} = \frac{\partial d_1}{\partial a} f(-d_1) - a e^{-bT} \frac{\partial d_2}{\partial a} f(-d_2) + e^{-bT} N(-d_2)$$

It is sufficient to use the symmetry of f(.) with respect to the origin to get the desired result.

#### Proof of proposition 3

When  $\theta < 1$ , the result is a direct consequence of proposition 2 because the denominator doesn't depend on  $\theta$ .

When  $\theta \ge 1$ , we write, using equation (9):

$$\alpha = \frac{(1 - \theta e^{-rT})}{C(\theta, r)}$$

$$\frac{\partial \alpha}{\partial \theta} = \frac{-e^{-rT}C(\theta, r) - (1 - \theta e^{-rT})\frac{\partial C}{\partial \theta}(\theta, r)}{C(\theta, r)^2}$$

Applying lemma 2 leads to:

$$\frac{\partial \alpha}{\partial \theta} = \frac{-e^{-rT}C(\theta, r) + (1 - \theta e^{-rT})e^{-rT}N(d_2(\theta, r))}{C(\theta, r)^2}$$

We know that  $C(\theta, r) = N(d_1(\theta, r)) - \theta e^{-rT} N(d_2(\theta, r))$ . It simplifies the preceding expression in:

$$\frac{\partial \alpha}{\partial \theta} = \frac{-e^{-rT}N(d_1(\theta, r)) + e^{-rT}N(d_2(\theta, r))}{C(\theta, r)^2}$$

$$= e^{-rT}\frac{N(d_2(\theta, r)) - N(d_1(\theta, r))}{C(\theta, r)^2}$$
(26)

#### Proof of proposition 4

The expression of F given in equation (16) allows to express the derivative of F with respect to  $\theta$  in a simple way.

$$\frac{\partial F}{\partial \theta} = e^{\mu T} \frac{\partial P}{\partial \theta}(\theta, \mu) - \omega \frac{\partial P}{\partial \theta}(\theta, r)$$

with

$$\omega = \lambda e^{\mu T} \frac{C(1,\mu)}{C(1,r)}$$

Lemma 2 provides the derivative of the put prices and implies directly the first result in proposition 4:

$$\frac{\partial F}{\partial \theta} = N(-d_2(\theta, \mu)) - \omega e^{-rT} N(-d_2(\theta, r))$$

By equation (16),  $F(\theta)$  can also be written:

$$F(\theta) = e^{\mu T} \left[ \frac{P(\theta, \mu)C(1, r) - \lambda C(1, \mu)P(\theta, r)}{C(1, r)} \right]$$

But  $r < \mu$  implies  $P(\theta, \mu) < P(\theta, r)$  and  $C(1, r) < C(1, \mu)$  because a call (put) price is an increasing (decreasing) function of the interest rate. Therefore,  $F(\theta) < 0$ as far as  $\lambda > 1$ .

Concerning the sign of the derivative of F with respect to  $\theta$ , we can write:

$$-d_2(\theta,\mu) = \frac{\ln(\theta) - (\mu - \sigma^2/2)T}{\sigma\sqrt{T}} < \frac{\ln(\theta) - (r - \sigma^2/2)T}{\sigma\sqrt{T}} = -d_2(\theta,r)$$

This inequality, combined with  $\lambda > 1, \mu > r$  and  $C(1, \mu) > C(1, r)$  show that  $\partial F / \partial \theta < 0.$ 

#### Proof of lemma 5

$$\begin{split} \int_{p}^{q} (z-x)g(x)dx &= \frac{1}{\sigma\sqrt{2\pi T}} \int_{p}^{q} (z-x)\frac{1}{x} \exp\left(-\frac{1}{2}\left(\frac{\ln(x) - (\mu - \sigma^{2}/2)T}{\sigma\sqrt{T}}\right)^{2}\right)dx \\ &= \frac{z}{\sqrt{2\pi}} \int_{-d_{2}(p,\mu)}^{-d_{2}(q,\mu)} \exp\left(-\frac{1}{2}y^{2}\right)dy \\ &- \frac{1}{\sigma\sqrt{2\pi}} \int_{\ln(p)}^{\ln(q)} \exp\left(-\frac{1}{2}\left(\frac{y - (\mu - \sigma^{2}/2)T}{\sigma\sqrt{T}}\right)^{2} + y\right)dy \\ &= \frac{z}{\sqrt{2\pi}} \int_{-d_{2}(p,\mu)}^{-d_{2}(q,\mu)} \exp\left(-\frac{1}{2}y^{2}\right)dy \\ &- \frac{e^{\mu T}}{\sigma\sqrt{2\pi}} \int_{\ln(p)}^{\ln(q)} \exp\left(-\frac{1}{2}\left(\frac{y - (\mu + \sigma^{2}/2)T}{\sigma\sqrt{T}}\right)^{2}\right)dy \\ &= \frac{z}{\sqrt{2\pi}} \int_{-d_{2}(p,\mu)}^{-d_{2}(q,\mu)} \exp\left(-\frac{1}{2}y^{2}\right)dy \\ &= \frac{z}{\sqrt{2\pi}} \int_{-d_{2}(p,\mu)}^{-d_{1}(q,\mu)} \exp\left(-\frac{1}{2}y^{2}\right)dy \\ &= z\left[N(-d_{2}(q,\mu)) - N(-d_{2}(p,\mu))\right] \\ &= e^{\mu T}\left[N\left(-d_{1}(q,\mu)\right) - N(-d_{1}(p,\mu))\right] \end{split}$$

When z = q, we know that:

$$P(q,\mu) = q e^{-\mu T} N(-d_2(q,\mu)) - N(-d_1(q,\mu))$$
$$P(p,\mu) = p e^{-\mu T} N(-d_2(p,\mu)) - N(-d_1(p,\mu))$$

It implies:

$$e^{\mu T} \left( P(q,\mu) - P(p,\mu) \right) = q N(-d_2(q,\mu)) - e^{\mu T} N(-d_1(q,\mu)) - p N(-d_2(p,\mu)) + e^{\mu T} N(-d_1(p,\mu))$$
  
$$= q \left[ N(-d_2(q,\mu)) - N(-d_2(p,\mu)) \right] + (q-p) N(-d_2(p,\mu))$$
  
$$-e^{\mu T} \left( N(-d_1(q,\mu)) - N(-d_1(p,\mu)) \right)$$

Therefore:

$$\int_{p}^{q} (q-x)g(x)dx = e^{\mu T} \left( P(q,\mu) - P(p,\mu) \right) - (q-p)N(-d_2(p,\mu))$$

#### Proof of proposition 6

The derivative of G with respect to  $\theta$  is given by:

$$\frac{\partial G}{\partial \theta} = e^{\mu T} \left[ -\frac{C(1,\mu)}{C(1,0)} \frac{\partial P(\theta,0)}{\partial \theta} + \lambda \frac{\partial P(\theta,\mu)}{\partial \theta} \right]$$
$$= e^{\mu T} \left[ -\frac{C(1,\mu)}{C(1,0)} N(-d_2(\theta,0)) + \lambda e^{-\mu T} N(-d_2(\theta,\mu)) \right]$$

We deduce that:

$$\frac{\partial G}{\partial \theta} = 0 \Leftrightarrow \frac{N(-d_2(\theta,\mu))}{N(-d_2(\theta,0))} = \frac{e^{\mu T}C(1,\mu)}{\lambda C(1,0)}$$
(27)

We also know that:

$$-d_2(\theta,\mu) = \frac{\ln(\theta) - \left(\mu - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} < -d_2(\theta,0)$$
(28)

Therefore

$$\frac{N(-d_2(\theta,\mu))}{N(-d_2(\theta,0))} < 1 < \frac{e^{\mu T}C(1,\mu)}{C(1,0)}$$

For  $\lambda$  sufficiently high, a stationary point for G may exist but it doesn't always correspond to a maximum and, in fact, it is a minimum here. To see this, we calculate the second derivative of G with respect to  $\theta$ .

$$\frac{\partial^2 G}{\partial \theta^2} = \frac{1}{\theta \sigma \sqrt{T}} \left( \lambda f(-d_2(\theta, \mu)) - \frac{e^{\mu T}}{C(1, r)} C(1, \mu) f(-d_2(\theta, 0)) \right)$$

At the stationary point satisfying  $\partial G/\partial \theta=0$  , we get:

$$\frac{\partial^2 G}{\partial \theta^2} = \frac{\lambda}{\theta \sigma \sqrt{T}} \left( f(-d_2(\theta, \mu)) - \frac{N(-d_2(\theta, \mu))}{N(-d_2(\theta, 0))} f(-d_2(\theta, 0)) \right)$$
$$= \frac{\lambda N(-d_2(\theta, \mu))}{\theta \sigma \sqrt{T}} \left( \frac{f(-d_2(\theta, \mu))}{N(-d_2(\theta, \mu))} - \frac{f(-d_2(\theta, 0))}{N(-d_2(\theta, 0))} \right)$$

Let us denote  $-d_2(\theta, \mu) = a$  and  $-d_2(\theta, 0) = b$ .

$$\frac{f(-d_2(\theta,\mu))}{N(-d_2(\theta,\mu))} - \frac{f(-d_2(\theta,0))}{N(-d_2(\theta,0))} > 0 \Leftrightarrow \frac{1}{\int_{-\infty}^a \exp\left(\frac{a^2 - t^2}{2}\right) dt} > \frac{1}{\int_{-\infty}^b \exp\left(\frac{b^2 - t^2}{2}\right) dt}$$

This inequality is then true because  $\int_{-\infty}^{y} \exp\left(\frac{y^2-t^2}{2}\right) dt$  is increasing in y and a < b, as shown in (28).

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Figure 1: Payoff of the note when  $S_0=1,\,\theta=0.9$  and  $\alpha=0.6$ 



Figure 2:  $\alpha$  versus  $\theta$  with  $r = 4\%, T = 1, \sigma = 20\%$ 



Figure 3: Gains and losses with the index as the reference point



Figure 4: Ratio of  $N(-d_2(\theta,\mu))/N(-d_2(\theta,r))$  as a function of  $\theta$ .  $\mu = 6\%; \sigma = 20\%, T = 1.$ 



Figure 5:  $G(\theta)$  as a function of  $\theta$  with  $\lambda = 2.25; \sigma = 0.2; T = 1; \mu = 0.06$ 





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