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Mixed Risk Aversion and Preference for Risk Disaggregation

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Abstract: In a recent paper entitled "Putting Risk in its Proper Place", Eeckhoudt and Schlesinger (2006) established a theorem linking the sign of the n-th derivative of an agent's utility function to her preferences among pairs of simple lotteries. We characterize these lotteries and show that, in a given pair, they only differ by their moments of order greater than or equal to n. When the n-th derivative of the utility function is positive (negative) and n is odd (even), the agent prefers a lottery with higher (lower) $n+2p$-th moments for p belonging to the set of positive integers. This result links the preference for disaggregation of risks across states of nature and the structure of moments preferred by mixed risk averse agents. It can be viewed as a generalization of a proposition appearing in Ekern (1980) which focused only on the differences in the n-th moments.

Keywords: risk apportionment, mixed risk aversion, prudence, temperance

JEL: D81

Introduction

Risk aversion, prudence and temperance have been extensively studied in the literature and they are mainly characterized in two ways. Some economists prefer to define these concepts by signing the derivatives of the utility function, others refer to behavioural traits. For example, prudence is shown to be linked to a precautionary savings motive by Kimball (1990), or, equivalently, as a "location preference" by Eeckhoudt *et al*.(1995). If a prudent agent has to bear a new risk but may choose to locate it on wealthy or less wealthy states of nature, he will always choose to locate it on the wealthy states. The other way to characterize prudence is the positive sign of the third derivative of the utility function. In the same vein, temperate agents dislike mutually aggravating risks (Kimball, 1993). When such an agent bears a risk $\tilde{\varepsilon}_1$ on a subset of states of nature Ω_1 and has to bear a new risk $\tilde{\varepsilon}_2$, she prefers to attach it, if possible, to the complementary set Ω_1^c.

For higher order preferences, Eeckhoudt and Schlesinger (2006) introduce risk apportionment of order *n* by imposing preferences among pairs of lotteries built through a nesting process. Their essential result establishes the link between this risk apportionment and the *n*-th derivative of the utility function.

In this paper, we first recall the process used by Eeckhoudt and Schlesinger (2006) to build the sequence of lottery pairs but we present this process in a more "probabilistic" way. It then makes easier the analysis of the moments of these lotteries and allows us to explain directly the structure of preferences among the pairs when agents exhibit mixed risk aversion.

A utility function *u* exhibits mixed risk aversion, as defined by Caballé and Pomansky (1996), if *u* is infinitely differentiable and the derivatives of *u* are alternating in sign, more precisely, $\operatorname{sgn}\left(u^{(n)}\right) = (-1)^{n+1}$ for any positive integer *n*. Using the terminology of Ekern (1980), we can say that the agent is *n*-th degree risk averse for any integer *n*.

This class of functions includes most of the commonly used utility functions as the negative exponential or the power utility functions. This assumption is then not too restrictive. We then build on Eeckoudt and Schlesinger (2006) who link the sign of the *n*-th derivative of *u* to the preferences among pairs of simple lotteries. Their main tool is the utility premium of a lottery which is the difference of expected utility obtained by the agent with and without the lottery. Our approach is, in a sense, more natural, since we use directly the successive moments of the lotteries to establish the preferences among pairs of lotteries.

These lotteries, denoted as (A_n, B_n), are obtained through a nesting process for $n \geq 3$ which is in the spirit of the sequence of "odd" and "even" lotteries appearing in proposition 3.1 of Caballé and Pomansky (1996).

For n = 3 and 4, Eeckhoudt and Schlesinger (2006) link their result to the concepts of prudence and temperance introduced by Miles Kimball (1990, 1992), and, for n = 5, to "edginess" (Fatima Lajeri-Chaherli, 2004). For orders larger than 5, they introduce the term "risk apportionment of order n" which means that agents prefer to disaggregate risk across states of nature.

Our purpose in this paper is to show that the moments of the lotteries (A_n, B_n) satisfy very peculiar properties. We first demonstrate that the moments of order $k < n$ of the two lotteries are equal for every n. We then show that the sign of the difference of the n-th moments depends on the parity of n. When n is odd(even), the mixed risk averse agent prefers the lottery characterized by the largest (lowest) n-th moment. These results may also be deduced of a paper by Ekern (1980); in this paper, the proof was based on "n-th degree risk increase" and used integration by parts.

Our results extend Ekern's proposition by showing that the moments of order $n + 2p$ (p being any positive integer) are equal in magnitude but have opposite signs when n is odd. When n is even, the lottery with the lower moments of order $(n + 2p)$ is preferred.

Our results are also interesting for experimental purposes. They allow to test if agents prefer low or high $(n + 2p)$-th moments when moments of order lower than n are equal. They can then be used to test the relevance of the mixed risk aversion assumption.

I. Notations and preliminary results

Denote $(\tilde{x},\tilde{y})$ a pair of lotteries, *u* a mixed risk averse utility function and *W* the initial wealth of the agent characterized by *u*. $\tilde{x}$ preferred to $\tilde{y}$ means:

$$E\left[u(W+\tilde{x})\right] \geq E\left[u(W+\tilde{y})\right] \tag{1}$$

Using the infinite Taylor series expansion of the two terms leads to:

$$\sum_{n=1}^{+\infty} u^{(n)}(W)E\left[\tilde{x}^n-\tilde{y}^n\right] \geq 0 \tag{2}$$

This inequality is obviously satisfied if for every *n* we have:

$$u^{(n)}(W)E\left[\tilde{x}^n-\tilde{y}^n\right] \geq 0 \tag{3}$$

u being mixed risk averse, the sign of $u^{(n)}(W)$ is $(-1)^{n+1}$. Inequality (3) is then satisfied if either the *n*-th moments of the two lotteries are equal or if

$$\operatorname{sgn}\left[E\left[\tilde{x}^n-\tilde{y}^n\right]\right]=(-1)^{n+1} \tag{4}$$

In the following sections, we will show that the pairs (A_n, B_n) defined hereafter satisfy such a property. It is worth to note that the Taylor series expansion in inequality (2) is not truncated. Therefore, we have to prove inequality (3) for every integer *n*. It is in fact well known that preferences for some moments are not sufficient to ensure preferences for a probability distribution over another one. For example, it is often argued that gamblers prefer positive skewness but preference for a larger third moment is in general not compatible with Expected Utility (EU) maximization, except if the utility function is cubic. Menezes et al. (1980) and more recently Shiu (2008) provide skewness comparability conditions under which EU maximization and a larger third moment are equivalent.

Let now $(\tilde{x},\tilde{y})$ denote a pair of random variables (possibly degenerate) and let $\tilde{z}=\left[\tilde{x},\tilde{y}\right]$ denote a compound lottery which takes the results $\tilde{x}$ or $\tilde{y}$ with equal probabilities. The simplified notation $\left[\tilde{x}\right]$ means that the result $\tilde{x}$ has a probability equal to 1. As an illustration, assume that $\tilde{x}$ and $\tilde{y}$ are binary lotteries respectively taking values (x_1, x_2) and (y_1, y_2). The realization of $\tilde{z}$ is obtained in two steps: first, a coin toss is realized to select either (x_1, x_2) or (y_1, y_2). Then, another draw is performed to get the first or the second outcome, according to the probability distribution of the variable drawn in the first step, $\tilde{x}$ or $\tilde{y}$.

We now recall the process used by Eeckhoudt and Schlesinger (2006) (ES in the following) to build a nested sequence of lotteries to link derivatives of utility functions to behavioral characteristics of agents. Remember that the usual concepts of monotonic preferences and risk aversion mean respectively that agents prefer more than less and that they are averse to mean preserving spreads. These basic concepts lead to the following definition for the two first pairs of lotteries.

1) $A_1 = [-k], A_2 = [\tilde{\varepsilon}_1], B_1 = B_2 = [0]$ where $\tilde{\varepsilon}_1$ is a binary random variable taking equally likely values $(\varepsilon_1; -\varepsilon_1)$, ε_1 and k being positive constants.

Monotonic preferences then imply that B_1 is preferred to A_1 and risk aversion means that B_2 is preferred to A_2.

2) Prudence is defined by ES as the preference of B_3 over A_3 where these lotteries are characterized by:

$$\begin{aligned} A_3 &= [0; \tilde{\varepsilon}_1 - k] \\ B_3 &= [-k; \tilde{\varepsilon}_1] \end{aligned} \tag{5}$$

Obviously, it is not the definition initially given by Kimball (1990) but ES show that the two coincide. The important point here is to remark that the difference between B_3 and A_3 is the « location » of the new risk $\tilde{\varepsilon}_1$. Prudent agents prefer not to add $\tilde{\varepsilon}_1$ on the « wealthy » state or, alternatively, they prefer a wealth reduction – k in the less risky state.

To introduce the nesting process in an intuitive way, remark that:

$$\begin{aligned} A_3 &= [0; \tilde{\varepsilon}_1 - k] = [B_1; A_1 + \tilde{\varepsilon}_1] \\ B_3 &= [-k; \tilde{\varepsilon}_1] = [A_1; B_1 + \tilde{\varepsilon}_1] \end{aligned} \tag{6}$$

A_3 is obtained by adding the new risk $\tilde{\varepsilon}_1$ to $[A_1; B_1]$ on the A lottery when B_3 is built by attaching $\tilde{\varepsilon}_1$ to the B lottery.

3) Temperance is now characterized by the preferences over lotteries A_4 and B_4 defined as follows:

$$\begin{aligned} A_4 &= \left[0; \tilde{\varepsilon}_1 + \tilde{\varepsilon}_2\right] = \left[B_2; A_2 + \tilde{\varepsilon}_2\right] \\ B_4 &= \left[\tilde{\varepsilon}_1; \tilde{\varepsilon}_2\right] = \left[A_2; B_2 + \tilde{\varepsilon}_2\right] \end{aligned} \tag{7}$$

where $\tilde{\varepsilon}_2$ is independent of $\tilde{\varepsilon}_1$ and takes two equally likely values $(\varepsilon_2; -\varepsilon_2)$. A temperate agent will prefer B_4 over A_4. She prefers to attach the new risk $\tilde{\varepsilon}_2$ to the less risky situation.

4) ES pursue the nesting process of lotteries and provide a general definition of the sequence (A_n, B_n) as follows:

5) For $n \geq 3$ $\begin{cases} A_n = \left[B_{n-2}; \tilde{\varepsilon}_{Int(n/2)} + A_{n-2}\right] \\ B_n = \left[A_{n-2}; \tilde{\varepsilon}_{Int(n/2)} + B_{n-2}\right] \end{cases}$ (8)

where $Int(x)$ denotes the largest integer lower than x. The variables $\tilde{\varepsilon}_j, j \geq 1$ are assumed to be independent and defined in the same way as $\tilde{\varepsilon}_1$.

Table 1 summarizes the definitions of the lotteries for $n \leq 4$. As mentioned before, when n =1, 2, preferring B to A means monotonicity and risk aversion. When n = 3, preferring B means that prudent agents prefer supplementary risks in wealthier states. When n = 4, the same kind of interpretation may be done with "less risky states" instead of "wealthier states". In other words, agents dislike mutually aggravating risks.

Table I around here

A_n and B_n are compound lotteries and their definition means that a first coin is tossed to select one of the two lotteries appearing between brackets. Then, a state of nature is revealed which gives a value to the lottery selected in the first toss.

II. Risk apportionment

Definition 1

Preferences are said to satisfy risk apportionment of order n if, for the lotteries (A_n, B_n) *defined above, the individual always prefer* B_n *to* A_n.

Risk apportionment then means that agents prefer to disaggregate risks across states of nature. The essential result of Louis Eeckhoudt and Harris Schlesinger (2006, pp 286) is the following.

Theorem 1

In an expected utility framework with differentiable u, risk apportionment of order n is equivalent to the condition sgn$\left[u^{(n)}\right] = (-1)^{n+1}$ *where sgn*(.) *is the sign function valued* 1 (-1) *if the argument is positive* (*negative*).

In their paper, ES provide some details about the interpretation of this theorem for *n* = 3, 4, 5, in terms of prudence, temperance and edginess. However, they do not study the properties of the nested sequence of lotteries (A_n, B_n), and especially they do not analyze the moments of these lotteries. Our purpose in this paper is to investigate the successive moments of A_n and B_n and to show that they are closely related. Moreover, they directly explain the preferences among lotteries for mixed risk averse agents.

A. The moments of A_n and B_n for *n* = 3.

To provide an intuition of our general results, we first consider the case *n* = 3. A_3 and B_3 are defined as follows:

$$\begin{cases} A_3 = \left[B_1; \tilde{\varepsilon}_1 + A_1\right] = \left[0; \tilde{\varepsilon}_1 - k\right] \\ B_3 = \left[A_1; \tilde{\varepsilon}_1 + B_1\right] = \left[-k; \tilde{\varepsilon}_1\right] \end{cases} \tag{9}$$

Getting the final values of A_3 and B_3 implies two coin tosses. The first one specifies the random variable to be observed, the second one gives the observation of this variable. For example, if the first toss selects $\tilde{\varepsilon}_1 - k$ for A_3, the second one gives $\varepsilon_1 - k$ or $-\varepsilon_1 - k$. Obviously, if 0 is selected at the first toss, the second toss also gives this value. Strictly speaking, three states of nature are sufficient to describe the process but it is more comfortable to consider 2^{n-1} states when studying (A_n, B_n). In other words, (A_3, B_3) can be written as:

$$B_3 = \begin{bmatrix} -k \\ -k \\ \varepsilon_1 \\ -\varepsilon_1 \end{bmatrix} \text{ and } A_3 = \begin{bmatrix} 0 \\ 0 \\ \varepsilon_1 - k \\ -\varepsilon_1 - k \end{bmatrix} \tag{10}$$

The probability of each of the four values is then ¼.

The corresponding centered variables are:

$$B_3 - E(B_3) = \begin{bmatrix} -k/2 \\ -k/2 \\ \varepsilon_1 + k/2 \\ -\varepsilon_1 + k/2 \end{bmatrix} \text{ and } A_3 - E(A_3) = \begin{bmatrix} k/2 \\ k/2 \\ \varepsilon_1 - k/2 \\ -\varepsilon_1 - k/2 \end{bmatrix} \tag{11}$$

Let $M_k(\tilde{x}) = E\left[\left(\tilde{x} - E(\tilde{x})\right)^k\right]$ denote the *k*-th central moment of a random variable $\tilde{x}$. For $n = 3$, we get:

$$\begin{aligned} M_1(A_3) &= M_1(B_3) = 0 \\ M_2(A_3) &= M_2(B_3) = \frac{\varepsilon_1^2}{2} + \frac{k^2}{4} \\ M_3(A_3) &= -M_3(B_3) = -\frac{3\varepsilon_1^2 k}{4} \end{aligned} \tag{12}$$

The two lotteries differ by their 3rd moments which are equal in magnitude but exhibit opposite signs. More precisely, the preferred lottery has a positive skewness. The definition of the lotteries for $n = 3$ also implies that all odd central moments have opposite signs because $B_3 - E(B_3)$ and $-A_3 + E(A_3)$ take the same values (except that the two last values are exchanged but it has no consequence on moments). In fact, we can write:

$$\begin{aligned} M_n(B_3) &= \frac{1}{4}\left[2(-1)^n\left(\frac{k}{2}\right)^n + \left(\varepsilon + \frac{k}{2}\right)^n + (-1)^n\left(\varepsilon - \frac{k}{2}\right)^n\right] \\ M_n(A_3) &= \frac{1}{4}\left[2\left(\frac{k}{2}\right)^n + \left(\varepsilon - \frac{k}{2}\right)^n + (-1)^n\left(\varepsilon + \frac{k}{2}\right)^n\right] \end{aligned} \tag{13}$$

When n is even, $\left(-1\right)^n$ = 1 and the two moments are equal. If n is odd, we can use the convexity of the function $x \to x^n$ to show that $M_n(B_3) > 0$. Rearranging $M_n(B_3)$ leads to:

$$2M_n(B_3) = \frac{1}{2}\left[(\frac{k}{2}-\varepsilon)^n + (\frac{k}{2}+\varepsilon)^n \right] - \left(\frac{k}{2}\right)^n \qquad (14)$$

and the mean of $(\frac{k}{2}-\varepsilon)^n$ and $(\frac{k}{2}+\varepsilon)^n$ is greater than $\left(\frac{k}{2}\right)^n$.

B. The moments of A_n and B_n for *n* = 4.

For n = 4, the definition of (A_n, B_n) leads to:

$$\begin{cases} A_4 = \left[B_2; \tilde{\varepsilon}_2 + A_2\right] = \left[0; \tilde{\varepsilon}_2 + \tilde{\varepsilon}_1\right] \\ B_4 = \left[A_2; \tilde{\varepsilon}_2 + B_2\right] = \left[\tilde{\varepsilon}_1; \tilde{\varepsilon}_2\right] \end{cases} \qquad (15)$$

We use now 8 equally likely states of nature to characterize (A_4, B_4) .

$$B_4 = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_1 \\ -\varepsilon_1 \\ -\varepsilon_1 \\ \varepsilon_2 \\ -\varepsilon_2 \\ \varepsilon_2 \\ -\varepsilon_2 \end{bmatrix} \text{ and } A_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \varepsilon_2 + \varepsilon_1 \\ -\varepsilon_2 + \varepsilon_1 \\ \varepsilon_2 - \varepsilon_1 \\ -\varepsilon_2 - \varepsilon_1 \end{bmatrix} \qquad (16)$$

The two variables already have zero means. Simple calculations give the four first moments.

$$\begin{aligned} &M_1(A_4) = M_1(B_4) = 0 \\ &M_2(A_4) = M_2(B_4) = \frac{1}{2}\left(\varepsilon_1^{\,2} + \varepsilon_2^{\,2}\right) \\ &M_3(A_4) = M_3(B_4) = 0 \\ &M_4(A_4) = M_4(B_4) + 3\varepsilon_1^{\,2}\varepsilon_2^{\,2} = \frac{1}{2}\left(\varepsilon_1^{\,4} + 6\varepsilon_1^{\,2}\varepsilon_2^{\,2} + \varepsilon_2^{\,4}\right) \end{aligned} \qquad (17)$$

The first (non central) moment of A_4 and B_4 is 0. This property is preserved for moments of higher odd orders. In fact we have, for any integer j:

$$
\begin{aligned}
M_j(B_4) &= \frac{1}{4}\left(\left(\varepsilon_1{}^j + (-1)^j\,\varepsilon_1^j\right) + \left(\varepsilon_2{}^j + (-1)^j\,\varepsilon_2^j\right)\right) \\
M_j(A_4) &= \frac{1}{8}\left(\left(\varepsilon_1 + \varepsilon_2\right)^j\left(1+(-1)^j\right) + \left(\varepsilon_2 - \varepsilon_1\right)^j\left(1+(-1)^j\right)\right)
\end{aligned}
\tag{18}
$$

When j is odd, the two moments are zero and when j is even,

$$
\begin{aligned}
M_j(B_4) &= \frac{1}{2}\left(\varepsilon_1{}^j + \varepsilon_2{}^j\right) \\
M_j(A_4) &= \frac{1}{4}\left(\left(\varepsilon_1 + \varepsilon_2\right)^j + \left(\varepsilon_2 - \varepsilon_1\right)^j\right)
\end{aligned}
\tag{19}
$$

The convexity of the function $x \to x^j$ also implies here that $M_j(A_4) > M_j(B_4)$.

The following section provides a proof of the same kind of results for any positive integer *n*.

III. The general result

The analysis of the moments of A_n and B_n is divided in two parts, distinguished by the parity of *n*. It is justified by the fact that the first moments are 0 (different from 0) when *n* is even (odd).

Proposition 1

Let $n \geq 4$ even:

1) $M_j(A_n) = M_j(B_n)$ for $2 < j < n$

2) $\forall j$ odd, $M_j(A_n) = M_j(B_n) = 0$

3) For $p > 0$, $M_{n+2p}(A_n) > M_{n+2p}(B_n)$

Proof

A_2 and B_2 are zero-mean random variables. Since A_n and B_n are built from A_{n-2} and B_{n-2} by adding a zero-mean risk, all the variables A_n and B_n are also zero-mean variables. Central moments and non-central moments are then equal. The following lemma (the proof is reported in the appendix) will be useful to prove proposition 1. It is based on the following relationships where $m = Int(n/2)$:

$$
\begin{aligned}
E\left[A_n^j\right] &= \frac{1}{2}E\left[B_{n-2}^j\right] + \frac{1}{2}E\left[\left(\tilde{\varepsilon}_m + A_{n-2}\right)^j\right] \\
E\left[B_n^j\right] &= \frac{1}{2}E\left[A_{n-2}^j\right] + \frac{1}{2}E\left[\left(\tilde{\varepsilon}_m + B_{n-2}\right)^j\right]
\end{aligned}
\tag{20}
$$

Lemma 1

$$\forall j>0 \ \textit{and} \ \textit{even} \ n\geq 4, \ E\left[A_n^j\right]-E\left[B_n^j\right]= \ \frac{1}{2}\sum_{p=1}^{Int(j/2)}\binom{j}{2p}\varepsilon_m^{2p}\left(E\left[A_{n-2}^{j-2p}\right]-E\left[B_{n-2}^{j-2p}\right]\right)$$

Due to the results of the preceding section, we already know that:

$$\begin{aligned} &M_j\left[B_1\right]=M_j\left[B_2\right]=0 \\ &\forall\ j \text{ odd}, M_j\left[A_2\right]=0 \\ &\forall\ j>0 \text{ even}, M_j\left[A_2\right]=\varepsilon_1^j \end{aligned} \tag{21}$$

1) Consider first n = 4. Lemma 1 implies:

$$E\left[A_4^j\right]-E\left[B_4^j\right]= \ \frac{1}{2}\sum_{p=1}^{Int(j/2)}\binom{j}{2p}\varepsilon_2^{2p}\left(E\left[A_2^{j-2p}\right]-E\left[B_2^{j-2p}\right]\right) \tag{22}$$

For $j=3$, the sum contains only one term, equal to 0. For j > 3 odd and n = 4, the result is also true due to Eq. (21) and lemma 1. Iterating the process, we get that odd moments of A_n and B_n are equal for every even *n*.

When j is even lower than n, the reasoning takes the same kind of road. From section II we know that the result is true for n = 4. For n = 6 we get by lemma 1:

$$E\left[A_6^j\right]-E\left[B_6^j\right]= \ \frac{1}{2}\sum_{p=1}^{3}\binom{j}{2p}\varepsilon_3^{2p}\left(E\left[A_4^{j-2p}\right]-E\left[B_4^{j-2p}\right]\right) \tag{23}$$

The terms on the RHS are equal to zero because $j-2p$ is strictly lower than 4. It is then sufficient to increase n by steps of 2 to get the general result.

2) Let j denote a given odd number:

$$\forall j \text{ odd}, M_j\left[A_2\right]=M_j\left[B_2\right]=0 \tag{24}$$

This equality comes from results in section II.

We now show that if odd moments are 0 up to n-2, they are also 0 for n.

The definition of A_n and B_n leads to:

$$\begin{aligned} E\left[A_n^j\right]&=\frac{1}{2}E\left[B_{n-2}^j\right]+\ \frac{1}{2}\sum_{p'=0}^{Int(j/2)}\binom{j}{2p'}\varepsilon_m^{2p'}E\left[A_{n-2}^{j-2p'}\right] \\ &=\frac{1}{2}\left(E\left[B_{n-2}^j\right]+E\left[A_{n-2}^j\right]\right)+\ \frac{1}{2}\sum_{p'=1}^{Int(j/2)}\binom{j}{2p'}\varepsilon_m^{2p'}E\left[A_{n-2}^{j-2p'}\right] \end{aligned} \tag{25}$$

$$
\begin{aligned}
E\left[B_n^j\right] &= \frac{1}{2}E\left[A_{n-2}^j\right] + \frac{1}{2}\sum_{p'=0}^{Int(j/2)}\binom{j}{2p'}\varepsilon_m^{2p'}E\left[B_{n-2}^{j-2p'}\right] \\
&= \frac{1}{2}\left(E\left[B_{n-2}^j\right] + E\left[A_{n-2}^j\right]\right) + \frac{1}{2}\sum_{p'=1}^{Int(j/2)}\binom{j}{2p'}\varepsilon_m^{2p'}E\left[B_{n-2}^{j-2p'}\right]
\end{aligned} \tag{26}
$$

Since odd moments are 0 up to *n* – 2, it is also true for *n* since *j*-2*p’* is always odd.

3) Consider now even moments of order $j \geq n$. We have to show that $E\left[A_n^j\right] - E\left[B_n^j\right] > 0$.

This is satisfied for *j* = 4 and *n* = 4, thanks to the results in section II. For *j* > 4, we write:

$$
E\left[A_4^j\right] - E\left[B_4^j\right] = \frac{1}{2}\sum_{p'=1}^{Int(j/2)}\binom{j}{2p'}\varepsilon_2^{2p'}\left(E\left[A_2^{j-2p'}\right] - E\left[B_2^{j-2p'}\right]\right) \tag{27}
$$

All the terms in the sum are positive (see Eq. (24)).

If we assume the result is true up to *n* – 2, the following relationship implies it is also true for *n.*

$$
E\left[A_n^j\right] - E\left[B_n^j\right] = \frac{1}{2}\sum_{p'=1}^{Int(j/2)}\binom{j}{2p'}\varepsilon_2^{2p'}\left(E\left[A_{n-2}^{j-2p'}\right] - E\left[B_{n-2}^{j-2p'}\right]\right) \tag{28}
$$

We analyze now the case of “odd” lotteries.

Proposition 2

Let $n \geq 3$ odd:

1) $M_j(A_n) = M_j(B_n)$ for $2 < j < n$
2) $\forall j \geq 3$ odd, $M_j(A_n) = -M_j(B_n)$
3) For p odd, $M_{n+p}(A_n) > M_{n+p}(B_n)$

Proof

The essential difference between propositions 1 and 2 is that the variables A_n and B_n have a non-zero mean when *n* is odd. Their common expectation is – *k/*2. For example

$$
\begin{aligned}
E\left[A_3\right] &= \frac{1}{2}E\left[B_1\right] + \frac{1}{2}E\left[\tilde{\varepsilon}_1 + A_1\right] = -k/2 \\
E\left[B_3\right] &= \frac{1}{2}E\left[A_1\right] + \frac{1}{2}E\left[\tilde{\varepsilon}_1 + B_1\right] = -k/2
\end{aligned} \tag{29}
$$

For greater odd values of *n*, the expectations are the same because the $\tilde{\varepsilon}$ are zero-mean random variables.

1) The proof of proposition 2, dealing with central moments, is a little bit more involved than proposition 1 even if the formulation of the results is very close. The central moments can be decomposed in the following way.

$$\begin{aligned} M_j\left[A_n\right] &= E\left[(A_n+\frac{k}{2})^j\right] = \sum_{q=0}^{j}\binom{j}{q}\left(\frac{k}{2}\right)^q E\left[A_n^{j-q}\right] \\ M_j\left[B_n\right] &= E\left[(B_n+\frac{k}{2})^j\right] = \sum_{q=0}^{j}\binom{j}{q}\left(\frac{k}{2}\right)^q E\left[B_n^{j-q}\right] \end{aligned} \tag{30}$$

It implies that the difference of the two moments is:

$$M_j\left[A_n\right]-M_j\left[B_n\right]=\sum_{q=0}^{j-1}\binom{j}{q}\left(\frac{k}{2}\right)^q\left(E\left[A_n^{j-q}\right]-E\left[B_n^{j-q}\right]\right) \tag{31}$$

When $n = 3$ and $j < n$, the RHS of Eq. (31) is 0 thanks to the results in section II. In the general case, we assume that $M_j\left[A_{n-2}\right]=M_j\left[B_{n-2}\right]$ for $j < n - 2$ and we prove that $M_j\left[A_n\right]=M_j\left[B_n\right]$ for $j < n$. The following lemma will be useful for that purpose (the proof is reported in the appendix).

Lemma 2

For $p < n - 2$ and $j < n$, we have the relationship:

$$M_p\left[A_{n-2}\right]=M_p\left[B_{n-2}\right]\Rightarrow M_j\left[A_n\right]-M_j\left[B_n\right]=E\left[A_n^j\right]-E\left[B_n^j\right] \tag{32}$$

This lemma shows that it is sufficient to work with non central moments and to prove that $E\left[A_n^j\right]=E\left[B_n^j\right]$. We now use the definition of the lotteries to formulate the difference of moments, as in lemma 1.

$$E\left[A_n^j\right]-E\left[B_n^j\right]=\frac{1}{2}\sum_{p=1}^{j}\binom{j}{p}E\left[\varepsilon_m^p\right]\left(E\left[A_{n-2}^{j-p}\right]-E\left[B_{n-2}^{j-p}\right]\right) \tag{33}$$

As $E\left[\varepsilon_m\right]=0$, the first non-zero term in the RHS corresponds to $p = 2$. We then see that the RHS is equal to 0 when $n = 3$. Iterating the process, we obtain:

$$E\left[A_5^j\right]-E\left[B_5^j\right]=\frac{1}{2}\sum_{p=2}^{j}\binom{j}{p}E\left[\varepsilon_2^p\right]\left(E\left[A_3^{j-p}\right]-E\left[B_3^{j-p}\right]\right) \tag{34}$$

But the RHS is one more time equal to 0 because $j - p < n - 2$. The following iterations lead to the same result.

3) The following lemma shows that one of the relationships used with *n* even is also true when *n* is odd.

Lemma 3

For n odd and m = Int(n/2 *), we have:*

$$M_j\left[A_n\right]=\frac{1}{2}M_j\left[B_{n-2}\right]+\frac{1}{2}M_j\left[A_{n-2}+\tilde{\varepsilon}_m\right] \tag{35}$$

The proof is reported in the appendix.

From section II, we know that $M_j\left[A_3\right]=-M_j\left[B_3\right]$ for *j* odd. Assume that it is true up to *n* – 2. Lemma 3 allows to write:

$$M_j\left[A_n\right]+M_j\left[B_n\right]=\frac{1}{2}\left(M_j\left[A_{n-2}\right]+M_j\left[B_{n-2}\right]\right)+\frac{1}{2}\left(M_j\left[A_{n-2}+\tilde{\varepsilon}_m\right]+M_j\left[B_{n-2}+\tilde{\varepsilon}_m\right]\right) \tag{36}$$

The first term on the RHS is equal to 0 since it refers to the index *n* – 2. Moreover

$$M_j\left[A_{n-2}+\tilde{\varepsilon}_m\right]+M_j\left[B_{n-2}+\tilde{\varepsilon}_m\right]=\frac{1}{2}\sum_{p=1}^{j}\binom{j}{p}E\left[\varepsilon_m^p\right]\left(M_{j-p}\left[A_{n-2}\right]+M_{j-p}\left[B_{n-2}\right]\right) \tag{37}$$

But for odd values of *p*, $E\left[\varepsilon_m^p\right]=0$ and when *p* is even $M_{j-p}\left[A_{n-2}\right]+M_{j-p}\left[B_{n-2}\right]=0$ because *j* – *p* is odd.

4) We now prove the inequality between even moments of A_n and B_n

$$M_j\left[A_n\right]-M_j\left[B_n\right]=\frac{1}{2}\sum_{p=1}^{j}\binom{j}{p}E\left[\varepsilon_m^p\right]\left(M_{j-p}\left[A_{n-2}\right]-M_{j-p}\left[B_{n-2}\right]\right) \tag{38}$$

From section II, we kow that $M_j\left[A_2\right]-M_j\left[B_2\right]>0$. The above equation implies this inequality is also satisfied for *n* = 4. It is then sufficient to iterate the reasoning to get the result.

Table II around here

IV Concluding remarks

Table 2 summarizes the results contained in propositions 1 and 2. These two propositions share some common features, especially in points (1) and (3). The first point stipulates that the j – th central moments of the two lotteries A_n and B_n are equal for $j < n$. At order n, the difference between the moments depend on the parity of n. When n is even, the odd moments of A_n and B_n are equal to zero and when n is odd, the moments are equal in magnitude but exhibit opposite signs. Finally, when $j > n$, the j-th moments are equal to 0 when n is even and have opposite signs when n is odd. Concerning the economic interpretation of theorem 1, we observe that when the sign of the n-th derivative (n even) of the utility function is negative, the agent prefers the lottery with the lowest $n + 2p$-th moments. For example, if $n = 4$, it means that not only the kurtosis defines the preference over the pair of lotteries but all the even moments greater or equal to 4. When n is odd, the prudent agent prefers higher $n + 2p$-th moments. These results give some insights to test preferences in experimental settings.

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APPENDIX

Proof of lemma 1

For each even *n*, the central moments of A_n and B_n may be written as:

$$\begin{aligned} M_j\left[A_n\right] &= \frac{1}{2}M_j\left[B_{n-2}\right]+\frac{1}{2}M_j\left[\tilde{\varepsilon}_{Int(n/2)}+A_{n-2}\right] \\ M_j\left[B_n\right] &= \frac{1}{2}M_j\left[A_{n-2}\right]+\frac{1}{2}M_j\left[\tilde{\varepsilon}_{Int(n/2)}+B_{n-2}\right] \end{aligned} \tag{A1}$$

To simplify the notations, let us denote $m = Int(n/2)$.

$$\begin{aligned} M_j\left[\tilde{\varepsilon}_m+A_{n-2}\right] &= E\left[\left(\tilde{\varepsilon}_m+A_{n-2}\right)^j\right] \\ &= \sum_{p=0}^{j}\binom{j}{p}E\left[\tilde{\varepsilon}_m^{\,p}\right]E\left[A_{n-2}^{j-p}\right] \\ &= M_j\left[A_{n-2}\right]+\sum_{p=1}^{j}\binom{j}{p}E\left[\tilde{\varepsilon}_m^{\,p}\right]E\left[A_{n-2}^{j-p}\right] \end{aligned}$$

The product of expectations comes from the independence of the variables $\tilde{\varepsilon}$. However, odd moments of these variables are equal to 0. The moment of order 2*p* of $\tilde{\varepsilon}_m$ is equal to ε_m^{2p} because of the definition of this variable. We can then write:

$$M_j\left[\tilde{\varepsilon}_m+A_{n-2}\right] = M_j\left[A_{n-2}\right]+\sum_{p=1}^{Int(j/2)}\binom{j}{2p}\varepsilon_m^{2p}M_{j-2p}\left[A_{n-2}\right] \tag{A2}$$

The same calculations applied to *B* give:

$$M_j\left[\tilde{\varepsilon}_m+B_{n-2}\right] = M_j\left[B_{n-2}\right]+\sum_{p=1}^{Int(j/2)}\binom{j}{2p}\varepsilon_m^{2p}M_{j-2p}\left[B_{n-2}\right] \tag{A3}$$

Replacing expressions (A2) and (A3) in (A1) leads directly to the result and ends the proof of lemma 1.

Proof of lemma 2

We know by Eq. (9) that:

$$M_j\left[A_n\right]-M_j\left[B_n\right]=\sum_{q=0}^{j-1}\binom{j}{q}\left(\frac{k}{2}\right)^q\left(E\left[A_n^{j-q}\right]-E\left[B_n^{j-q}\right]\right)$$

At the same time, the difference $E\left[A_n^{j-q}\right]-E\left[B_n^{j-q}\right]$ can be written in the following way:

$$E\left[A_n^{j-q}\right]-E\left[B_n^{j-q}\right]=\frac{1}{2}\sum_{p=1}^{j-q}\binom{j-q}{p}E\left[\varepsilon_m^p\right]\left(E\left[A_{n-2}^{j-q-p}\right]-E\left[B_{n-2}^{j-q-p}\right]\right) \tag{A4}$$

The assumption of the lemma implies that the terms on the RHS of (A5) are equal to 0 as soon as q is greater or equal to 1, that is to say $j - q - p < n - 2$. Eq. (9) is then reduced to:

$$M_j\left[A_n\right]-M_j\left[B_n\right]=E\left[A_n^j\right]-E\left[B_n^j\right]$$

Proof of lemma 3

$$M_j\left[A_n\right]=E\left[\left(A_n+\frac{k}{2}\right)^j\right]=\sum_{p=1}^{j}\binom{j}{p}\left(\frac{k}{2}\right)^p E\left[A_n^{j-p}\right]$$

The definition of the lotteries implies:

$$E\left[A_n^{j-p}\right]=\frac{1}{2}E\left(B_{n-2}^{j-p}\right)+\frac{1}{2}E\left[\left(A_{n-2}+\tilde{\varepsilon}_m\right)^{j-p}\right]$$

It follows that:

$$M_j\left[A_n\right]=\frac{1}{2}\left[\sum_{p=1}^{j}\binom{j}{p}\left(\frac{k}{2}\right)^p E\left(B_{n-2}^{j-p}\right)+\sum_{p=1}^{j}\binom{j}{p}\left(\frac{k}{2}\right)^p E\left[\left(A_{n-2}+\tilde{\varepsilon}_m\right)^{j-p}\right]\right]$$

and then:

$$M_j\left[A_n\right]=\frac{1}{2}\left[M_j\left[B_{n-2}\right]+M_j\left[A_{n-2}+\tilde{\varepsilon}_m\right]\right]$$

n	A	B
1	$[-k]$	$[0]$
2	$[\tilde{\varepsilon}_1]$	$[0]$
3	$[0; \tilde{\varepsilon}_1 - k]$	$[-k; \tilde{\varepsilon}_1]$
4	$[0; \tilde{\varepsilon}_1 + \tilde{\varepsilon}_2]$	$[\tilde{\varepsilon}_1; \tilde{\varepsilon}_2]$

Table I: Lotteries (A_n, B_n) for *n* = 1,..,4

	n even	n odd
$j < n$	$M_j\left[A_n\right] - M_j\left[B_n\right]$	
$j = n$	$M_j\left[A_n\right] > M_j\left[B_n\right]$	$M_j\left[A_n\right] = -M_j\left[B_n\right] < 0$
$j > n$ $\left\{\right.$ *odd*	$M_j\left[A_n\right] = M_j\left[B_n\right] = 0$	$M_j\left[A_n\right] = -\ M_j\left[B_n\right] < 0$
even	$M_j\left[A_n\right] => M_j\left[B_n\right]$	

Table II: Summary of results of propositions 1 and 2

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