Symmetric vs. Downside Risk: Does It Matter for Portfolio Choice?

Olga Bourachnikova & Nurmukhammad Yusupov

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Abstract

While symmetric measures of risk, such as variance, have been conventionally used in finance, downside risk measures are arguably more intuitive although computationally more complex to use. Opponents of symmetric risk measures suggest that investors use downside risk approach to investment decisions. In this paper, using French stock market data, we empirically test whether the two approaches to portfolio optimization produce significantly different outcomes. Our results suggest portfolio choice under downside risk and symmetric risk frameworks yield similar results. Our paper contributes to the ongoing debate on the relevance of symmetric vs. downside risk measures. JEL Code: G11

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1. Introduction

Risk has been at the core of finance theory from the very beginning. Early research efforts on how investors facing risk allocate their capital across different assets culminated in two groundbreaking papers, Markowitz (1952) and Roy (1952), that marked the emergence of finance as a separate discipline. The former suggested that variance be used as a proxy for risk while the latter recognized the importance of downside risk in the investor’s decision making. Mainly because of its computational convenience, variance, along with standard deviation, quickly became widely accepted as a measure of risk in the mainstream of finance literature.

Nevertheless, variance has been widely criticized for being symmetric in terms of upside and downside deviations. One problem is that variance treats both the returns above and below the expected return equally. Intuitively it makes more sense to punish the investor or fund manager for low returns and reward for high returns. Thus, variance minimization is counter-intuitive as it entails punishment both for low and high returns equally.

This debate led to new concepts based on the idea of downside risk. One of the celebrated outcomes of this debate is the "value at risk" that emerged as a distinct concept in finance in the 1980s. With the advancement of available computing power the appeal of using downside risk measures as a more realistic concept in comparison to symmetric risk measures has gained ground in recent years. Another reason of the rise of downside risk approach in finance is due to extensive growth of financial derivatives industry. A natural question to ask in this debate
is whether the two frameworks of financial decision making produce significantly different results. If not, then why bother using the more costly method? Surprisingly, empirical investigation of this issue has so far received very little attention in the literature. This paper aims to fill that gap by offering a comparative analysis of portfolio choices under the two risk concepts.

Using the data from the French stock market, we compare optimal portfolios of two different investors that use the downside and symmetric risk measures. While for a symmetric risk investor we employ the seminal mean-variance model of Markowitz (1952), for a downside risk investor we rely on the model of Shefrin and Statman (2000). Drawing on the results of Lopes (1987) and Kahneman and Tversky (1979), the concept of downside risk was employed by Shefrin and Statman (2000) as a building block of their Behavioural Portfolio Theory (BPT). A BPT-investor seeks to maximize expected return subject to the probability of ruin being no greater than a critical level. Shefrin and Statman (2000) claim that in contrast to CAPM, that uses symmetric variance to account for risk, in equilibrium investors hold a portfolio that resembles a combination of bonds and a lottery ticket. Thus, according to BPT, investors deviate from the optimal portfolio diversification à la Markowitz, and consider their portfolios as a pyramid of assets with riskless instruments in the bottom layer and risky equity in the top layer\(^1\).

In economic theory, the idea that agents perceive the goal of not losing to be superior to that of gaining dates back to the early works of John Von Neumann and John Meynard Keynes\(^2\). The problem in the original model of Roy (1952) was

\(^1\)This idea was earlier articulated by Friedman and Savage (1948), Fisher and Statman (1997) among others.

\(^2\)Von Neumann presented his first paper in 1926 at the University of Göttingen. Source:
minimization of the probability of loss below a given subsistence level. Following Roy (1952) a number of papers used risk measures that are better attuned to the intuitive notion that risk is associated with loss or with failure to reach the target return (Baumol, 1963; Arzac and Bawa, 1977; Bawa and Lindenberg, 1977; Fishburn, 1977). Markowitz (1991) acknowledges that semivariance, that accounts for the downside deviations only, is a "more plausible measure of risk". Tversky and Kahneman (1992) show that losses are weighted about twice as strongly as gains.

An experimental examination of people’s risk perception in financial context is offered in Unser (2000) which supports conceptualization of risk as the failure to obtain a certain level of return. More recently, Bertsimas et al (2004) suggest that investors base their decisions on measures of shortfall rather than on the variance of returns. More broadly, the loss aversion over the individual stock returns can help explain high and volatile stock returns observed in the market (Barberis and Huang, 2001) or the equity premium puzzle (Barberis et al, 2001).

Our paper is related to a small set of empirical papers that compare portfolio choices under downside and symmetric risk frameworks. Harlow (1991) and Alexander and Baptista (2002) demonstrate that if return distributions are normal, the difference between the optimal portfolio choices of a symmetric-risk and downside-risk investors will be small. Jarrow and Zhao (2006) show that when asset returns are nearly normally distributed investors using variance and lower partial moment to measure risk choose similar portfolios. When asset returns are


Keynes had a mathematical analysis similar to that of Roy (1952) in Chapters 26 and 29 of his 1921 book A Treatise on Probability. Source: Brady (1996).
non-normal with large left tails they obtain the opposite result. However, numerous empirical studies carried out for different markets and for different periods confirm that the real returns are not normal (Mandelbrot, 1963; Brenner, 1974; Jorion, 1988). In line with these empirical studies, our simulations are insensitive to return distributions.

By simulating portfolio choices under two risk concepts we find that the optimal portfolio constructed by a downside-risk-averse investor belongs to the mean-variance efficient frontier. This result implies that both symmetric and downside risk measures yield similar portfolio choices in equilibrium which contradicts the assertions of Shefrin and Statman’s (2000) BPT. Our results provide empirical evidence that variance is a plausible risk measure in financial decision making.

The rest of the paper is structured as follows. Section 2 discusses the portfolio theory of the downside risk investor in the context of the BPT in contrast to the symmetric risk investor in mean-variance framework. Section 3 describes the dataset used in this paper. Section 4 explains the methodology of our portfolio simulations. Section 5 presents the results and Section 6 concludes the paper.

2. The Model

To model the portfolio choice of a downside risk investor we rely on BPT developed by Shefrin and Statman (2000) that is based on downside risk concept. To define risk Shefrin and Statman (2000) draw on Roy’s (1952) concept of safety-first approach. According to this concept an investor is characterised by a subsistence level of wealth $s$ and she is considered "ruined" if her terminal wealth $\tilde{W}$ falls below this exogenously given level $s$. Thus, Roy’s agent seeks to minimize the
probability of failure \( P(\hat{W} < s) \). Telser (1955) goes one step further to introduce the acceptable level for the ruin probability \( \alpha \) such that the portfolio is considered "safe" if the probability of failure \( P(\hat{W} < s) \) does not exceed \( \alpha \). Arzac and Bawa (1977) extend the Telser’s model by adding expected wealth in the criteria of choice. More precisely, they consider an investor whose objective function depends on the expected terminal wealth and \( \alpha \). These results serve as the basis for Shefrin and Statman’s (2000) Behavioural Portfolio Theory a simplified exposition of which we present herein.

Consider an economy in contingent claims at date zero. There are \( n \) states of nature \( \omega_1, \ldots, \omega_n \) each occurring with the probability \( p_i, i = 1, \ldots, n \) respectively. At date one the payoff from the asset \( e_i \) is 1 if \( \omega_i \) occurs and 0 otherwise. The price of \( e_i \) is known and denoted by \( \pi_i \). Suppose states are ordered so that state prices per unit probability \( \frac{\pi_i}{p_i} \) are monotonically decreasing in \( i \).

At date zero, the investor chooses a portfolio composed of the contingent claims that maximizes her expected terminal wealth subject to her budget constraint. A mean-variance investor with a quadratic utility function thus solves the following program:

\[
\text{max} \sum p_i \left( W_i - \frac{b}{2} W_i^2 \right) \quad \text{s.t.} \quad \sum \pi_i W_i \leq W_0
\]

\(^{3}\)In Shefrin and Statman (2000) the objective probabilities of events are distorted by BPT agent. Here we consider a particular case with true probabilities. Because our goal is to determine the difference in portfolio composition due to different concepts of risk (symmetric and downside risk), this restriction does not affect the result. Indeed, the results of Shefrin and Statman (2000) hold for a general form of distortion (or weighting) function, which may as well be an simple identity.
The solution to this portfolio problem has the following form:

\[
W_i = \frac{1}{b} \left[ 1 - \frac{\sum \pi_j - bW_0}{\sum \frac{\pi_j^2/p_j}{\pi_j/p_j}} \right]
\]  

(2.1)

where \( b \) is some constant.

A behavioral investor at date one maximizes her expected terminal wealth subject to safety-first constraint in addition to the budget constraint, so the optimisation program of a BPT investor is defined as:

\[
\max E(\widetilde{W}) \quad \text{s.t.} \quad P(\widetilde{W} < A) \leq \alpha \quad \text{and} \quad \sum \pi_i W_i \leq W_0
\]

(2.2)

where \( A \) is the aspiration level and \( \alpha \) is the maximum probability of failure. Both \( A \) and \( \alpha \) are private characteristics of the investor, also called security parameters. Thus, the agent seeks to maximise the expected wealth in a particular set of portfolios that meet the security constraint. \( \widetilde{W} \) denotes the future wealth distribution. \( \widetilde{W} \) takes the value \( W_i, i = 1, \ldots, n \), if \( \omega_i \) occurs. Shefrin and Statman (2000) show that any solution \( W_1, \ldots, W_n \) to (2.2) has the following form: there is a subset \( B \) of states, including the \( n \)-th state \( \omega_n \) such that

\[
W_i = 0 \quad \text{for} \quad i \notin B
\]

\[
W_i = A \quad \text{for} \quad i \in B \setminus \{\omega_n\}
\]

\[
W_n = \frac{W_0 - A \sum_{i \in B \setminus \{\omega_n\}} \pi_i}{\pi_n}
\]

where \( W_0 \) denotes the investor’s wealth at date zero.
As a rule, analytical solution to the behavioral portfolio problem given by (2.2) may not exist. However, we can demonstrate the portfolio choice of a BPT investor with a simple numerical example from Shefrin and Statman (2000). To do so, let us consider an economy with 8 states of nature, arbitrary prices of which are given in the following table:

<table>
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<tr>
<th>Prices</th>
<th>0.37</th>
<th>0.19</th>
<th>0.12</th>
<th>0.09</th>
<th>0.07</th>
<th>0.06</th>
<th>0.05</th>
<th>0.04</th>
</tr>
</thead>
</table>

For the sake of simplicity let us suppose that the probability is uniformly distributed: \( p_1 = p_2 = \ldots = p_8 \). The distribution for the optimal portfolio as a function of realised state at date one given \( W_0 = 1 \) (the agent invests 1 at date 0) is shown in Fig.1 both for Markowitz and Shefrin-Statman investors. In this example, both have the same expected return on their portfolios, but the latter is also characterized with \( \alpha = 0.25 \) and \( A = 2 \). We observe that Markowitz investor invests in each individual asset to lower her risk while the Shefrin-Statman does not. In the Shefrin-Statman case, this payoff pattern can be described as the combination of payoffs from a portfolio consisting of a bond and a lottery ticket that pays off only in state 8.

Shefrin and Statman (2000) show that efficient solutions of the behavioral portfolio problem are typically non mean-variance efficient. The idea is that an investor with a given level investment capital who perceives risk as the downside
deviation from the expectation makes two distinct investment decisions: first, she seeks to create a portfolio satisfying the safety-first criteria for her level of capital at the lowest possible level of required investment, i.e. at the cheapest price, and, second, and if the budget constraint is not satiated she allocates the remaining investment capital on the asset with the highest expected payoff. In the example above, for her initial wealth $W_0 = 1$ the investor proceeds in two simultaneous steps: she starts out by investing in the assets with the lowest ratio $\frac{a_i}{p_i}$ in order to ensure terminal value of the portfolio at date 1 at the level $A = 2$ in 75% of states of nature. In our example, she invests in the 6 cheapest assets. This strategy enables her to meet the security constraint with the lowest cost. This is why the payoffs in states 1 and 2 are zero. Then, she invests all the rest of the initial wealth in the cheapest available asset.

Thus, the composition of the resulting optimal portfolio of a downside-risk investor differs from that of a mean-variance investor who simply allocates all of the capital on the portfolio with a minimum level of variance for a given level of expected return. This hypothesis is the object of our tests in this paper.

3. Data

To collect the data, we consider stocks that composed the SBF120 French Index over the period from June 2001 to June 2007. Stock price observations were obtained from the database maintained by Fininfo, a French financial market data provider. At the beginning of that period the index was composed of 119 assets (as of June 1, 2001). To preserve continuity, of these 119 assets we eliminated, the assets that were excluded from the index over the period under consideration
due to various reasons. We also eliminated assets with missing observations. As a result we were left with 71 assets in the final sample, as opposed to the initial 119, with daily observations over 1535 days.

Using these 71 stocks we compute 1534 daily stock returns defined as follows:

\[ R_{i,t} = \ln (P_{i,t} + D_{i,t}) - \ln P_{i,t-1} \]

where:
- \( R_{i,t} \) denotes the return of the asset \( i \) at the time of \( t \),
- \( P_{i,t} \) is the price of the asset \( i \) at the time of \( t \),
- \( D_{i,t} \) is the dividend (\( D_{i,t} = 0 \) if no dividends are paid at the date of \( t \)).

Main descriptive statistics are given in the Table 1. To test for normality we resort to the Jarque-Bera test which reveals strong departure from normality. A priori, non-normality suggests a possibility that the optimal portfolios under symmetric and downside risk measures may not be close.

Due to operational limits of the computing capacity that was available to us we could only work with investment portfolios composed of at most 15 equities. Although this technical restriction may raise a concern, there is vast body of recent literature that documents under-diversification in the portfolios of real investors (Blume and Friend, 1975; Benartzi, 2001; Benartzi and Thaler, 2001; Polkovenichenko, 2005). For example, according to Goetzmann and Kumar (2008), US investors hold on average 3 different assets in their portfolios. Thus our empirical investigation, while may have somewhat restricted normative valid-
ity for institutional investment portfolios, should be positively valid for individual investors.

Because the goal of our study is to compare the optimal portfolio à la Markowitz with those of a downside risk minimizing agent it is a critical assumption that in our setting a reasonably well diversified portfolio can be constructed. We define a well-diversified portfolio as one which generates at least 90% reduction of the variance relative to a 2-asset portfolio. This definition is consistent with that used in the research literature (Statman, 2004) and academic textbooks (Bodie et al (1999), p.203). Thus, for robustness reasons, following the methodology of Campbell et al (2001) we test whether a reasonably well diversification can be achieved with 15 stock holdings in a portfolio. We denote the number of assets in a portfolio by $n = 2, 3, ..., 71$. The assets are chosen randomly and enter the portfolio with the equal weights. For each value of $n$ we compute the average variance of 10,000 randomly constructed portfolios composed of $n$ assets. We obtain that in the market under consideration (71 assets) a portfolio composed of 15 assets can reach sufficiently high diversification level. Figure 2 shows the results of our test for various values of $n$.

Naturally, the best, i.e. the most diversified, portfolio is the one composed of all 71 assets. Its variance is equal to $1.5 \times 10^{-4}$, the minimum possible level in this market. Similarly, the worst level of diversification $3.3 \times 10^{-4}$ is obtained for a 2-asset portfolio. The difference is $1.8 \times 10^{-4}$. This difference falls when the number of assets in the portfolio increases. Table 2 reveals numerical schedule of
this effect: 35% of variance is reduced when the number of assets in the portfolio goes from 2 to 3. If the portfolio contains 15 assets the variance is reduced by over 90% in comparison to the least-diversification scenario.

It is important to note that the number of assets in the composition of a well diversified portfolio varies depending on the market and the time period under consideration. It is 20 for Bloomfield, Leftwich and Long (1977), 30 for Statman (1987), 120 for Statman (2003). In the market considered in this study we demonstrate, using the method suggested by Campbell and al. (2001) that a portfolio composed of 15 assets can be considered is well diversified.

4. Methodology

To run portfolio simulations we construct optimal portfolios à la Markowitz and à la Shefrin-Statman. We proceed in the following two steps: first, we estimate expected annual returns using bootstrap method and, second, construct the portfolios using the state space from the first step.

Step 1. From among the 71 stocks in our sample we randomly choose 15 assets. We also select a random interval within the period 1.06.2001—01.06.2007 of 250 consecutive days. Thus, we construct the following matrix of daily returns, where $A_i$ denotes an individual asset, each element $r_{i,j}$ denotes a daily return of
the asset $i$ at the date $j$:

\[
\begin{array}{cccc}
  A_1 & A_2 & \cdots & A_{15} \\
  r_{1,1} & r_{2,1} & \cdots & r_{15,1} \\
  r_{1,2} & r_{2,2} & \cdots & r_{15,2} \\
  \vdots & \vdots & \ddots & \vdots \\
  r_{1,1534} & r_{2,1534} & \cdots & r_{15,1534}
\end{array}
\]

We resort to the historical simulation method (Hull and White, 1998) to compute the expected annual returns. Namely, to simulate the expected returns at a future date, in a one year period after the date of the stock price observations for 15 random assets, we randomly select a 250-day period in our sample. This is based on our assumption that a year consists of 250 days. Once we have daily returns we employ bootstrap method to calculate the annual expected returns. We randomly pick a date within the chosen 250-day period, which gives us daily returns for 15 stocks under question. We repeat this procedure randomly within the chosen 250-day period 250 times. Then we take the sum of the 250 daily returns to compute the annual returns for each of the 15 chosen stocks.

In line with the general principles of the bootstrap methodology, we take the future uncertainty to be represented by 1,000 states of nature. For that, we repeat the bootstrap procedure 1,000 times to simulate 1,000 lines of future annual
returns for 15 assets. Eventually we construct the following matrix:

\[
R = \begin{array}{cccc}
A_1 & A_2 & \cdots & A_{15} \\
\omega_1 & r_{1,1} & r_{1,2} & \cdots & r_{1,15} \\
\omega_2 & r_{2,1} & r_{2,2} & \cdots & r_{2,15} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\omega_{1000} & r_{1000,1} & r_{1000,2} & \cdots & r_{1000,15}
\end{array}
\]

Where \( r_{c,d} \) denotes the annual return of the asset \( A_c \) if the state \( \omega_d \) occurs.

**Step 2.** To compute the optimal portfolios we need to solve the following optimization problems: minimization of portfolio’s variance given the expected returns for Markowitz portfolio and maximization of the portfolio’s expected return given the probability, \( \alpha \), of earning below a threshold value, \( r^* \), for Shefrin-Statman portfolio. In the Markowitz (1952) problem the program is to minimize the variance of the portfolio returns given the expected return \( E(\bar{r}) \) or maximize the expected return subject to an acceptable level of variance of returns \( Var \). This problem is well known to have a closed form solution.

In the Shefrin-Statman problem given the expected return \( E(\bar{r}) \) and probability \( P(\bar{r} < r^*) \) of earning less than \( r^* \) the program writes

\[
\max E(\bar{r}) \text{ s.t. } P(\bar{r} < r^*) \leq \alpha
\]

Due to absence of analytical solutions, solution to this problem requires numerical methods.

We consider 12 different cases for the Shefrin-Statman problem: \( r^* \in \{0; 0.05; 0, 1\} \) and \( \alpha \in \{0; 0,1; 0,2; 0,3\} \). With these 12 scenarios we construct the state-space
matrix from the previous step 140 times\textsuperscript{4}. Thus, we have repeated the matrix simulation 1680 times. For each matrix we construct the Markowitz (1952) portfolio frontier and the portfolio optimal for a downside risk investor who makes her choice within the Shefrin-Statman framework. We presume that the part of initial wealth invested in a single asset is equal to $\frac{k}{16}$, $k = 0, 1, ..., 15$ and we consider a sample of 1,00,000 portfolios. The latter value is due to our subjective judgement. It can be shown that the total number different portfolios is about 77 million. For each of 1,00,000 portfolios we verify if it meets the security constraint. The set of portfolios that meet the security constraint is called the security set. According to the Shefrin-Statman problem the optimal downside risk portfolios is the one that belongs to the security set and that allows to reach the maximum expected value of returns. Whenever such portfolio exists, i.e. meets the regularity conditions, we compute its standard deviation and expected return. These two values for each portfolio enable us to locate the portfolio in the traditional risk-return space. In each case we compare the two portfolios to identify if the downside risk portfolio is superior to the symmetric risk portfolio in the weak Pareto sense, that is provides an expected return higher than and lower or the same standard deviation. Whenever such portfolios exist they are denoted by $P_M$.

5. Results

Out of 1680 we obtain 651 cases where no optimal downside risk portfolio exists as none of those 1,00,000 portfolios under consideration meets the security constraint.

\textsuperscript{4}The choice of 140 repetitions is somewhat arbitrary. We stopped the trials sufficiently long after the results started showing identical values.
Results of our calculations are shown in Table 3. By $N_S$ we denote the number of the optimal downside risk portfolios constructed for each couple $(r^*, \alpha)$. The more demanding the investor is in terms of the individual asset’s characteristics, the fewer elements her set of security contains. For example, we observe that when the admissible probability of failure $\alpha$ increases (and subsistence level $r^*$ remains the same) the number of portfolios that meet the security constraint increases. Similarly, if $r^*$ increases (and $\alpha$ remains unchanged), the security set becomes smaller. When $\alpha = 0$, the investor wants to insurance in all states of nature and, if $r^* = 0$, recover her initial investment in all states of nature. In this case there are only 30 cases out of 140 in which the investor could reach her goal. When $\alpha = 0$ and $r^* = 0,05$ the investor seeks to reach at least 5% return regardless of the state of nature. Thus scenario is even more difficult: there are only 20 situations when it is possible. Finally, if $\alpha = 0$ and $r^* = 0,1$, the investor will be satisfied only if she earns 10% without any possibility of failure. The number of portfolios meeting this constraint is limited to only 14.

We denote the number of cases, where Markowitz investor selects an optimal portfolio different from the optimal downside risk portfolio, by $N_M$. We realise that there are only 7 cases among 1029 (only 0,68% of cases) in which the choice of a Markowitz’s investor does not coincide with that of a downside investor. Let us take a closer look at such a case. We illustrate a common case where $N_M = 0$, which implies that the downside investor chooses a portfolio at the Markowitz frontier.
Figure 3 depicts all 1,00,000 portfolios that are characterized by their standard deviation (on the horizontal axis) and their expected return (on the vertical axis). Each blue point represents one portfolio of 15 individual assets.

**INSERT FIGURE 3 ABOUT HERE!**

In Figure 4, we depict using green dots all the individual securities that satisfy the security constraint of a downside-averse investor with $r^* = 0$ and $\alpha = 0, 3$, meaning that the investor’s aspiration level equals the initial investment and the admissible level of probability of generating negative returns is 30%.

**INSERT FIGURE 4 ABOUT HERE!**

Thus, all green portfolios allow the investor to recover his initial investment in 70% of the cases. The optimal portfolio will be the one that provides the highest return given that constraint. We can see that it belongs to the Markowitz efficient frontier as do all the portfolios on the upper edge of the graph. In this particular case, it may seem that this result is obtained because the security set is sufficiently large and includes the Markowitz efficient frontier.

However, in the following our data reveal that this result holds for various levels of aspiration and admissible failure. Imagine an investor, who is more risk-averse. Naturally, her security set will contain fewer portfolios. Consider the following values of $\alpha = 20, 10$ and 0 percent. These cases are represented in Figure 5. The set of purple (pink) points identifies portfolios that allow the investor to recover his initial investment in any state of nature, i.e. that create perfect insurance against any loss. The set of light blue portfolios, which includes the set of purple
(pink) portfolios, allows the investor to recover his initial investment in 90% of cases; the red set is for an investor whose $\alpha$ equals 80%; the red set includes the purple (pink) and light blue sets and so on.

**INSERT FIGURE 5 ABOUT HERE!**

We find the same result when $r^*$ increases and $\alpha$ remains at the same level (see Figure 6).

**INSERT FIGURE 6 ABOUT HERE!**

Thus, we can conclude that for any level of $\alpha$ and $r^*$ the security set will always contain a part of the efficient Markowitz frontier. This is what is meant when we suggest that the optimal downside risk portfolios coincide with those of a Markowitz investor.

In Figure 5, the letter B represents the portfolio with the highest return out of all the light blue portfolios. Thus, B is the optimal portfolio for a BPT investor characterised by an aspiration level equal to the initial investment and an $\alpha$ of 90%. At the same time, B is optimal in the Markowitz (1952) sense for an investor who requires the level of risk which corresponds to 0.18 of standard deviation. The same reasoning applies to the portfolio represented by the letter A. It is optimal under BPT ($\alpha = 0, r^* = 0$) and also in the Markowitz sense.

We notice two interesting points. The first one concerns the measures of risk. In both cases, the Markowitz approach and the BPT, we note that the more risk-averse is the investor the less risky is her optimal portfolio. Indeed, under BPT, an investor who requires more security will build up a less risky portfolio. Less risky not only in terms of downside risk measure but also in terms of Markowitz
framework. The optimal portfolio of an investor who requires more security is on the left of the Markowitz efficient frontier. Both portfolio A and B are on the efficient Markowitz frontier. A is less risky than B in both criteria.

The second interesting point concerns a specific case when $\alpha$ is equal to 0. Consider the set of purple (pink) portfolios (Figure 5). Here, loss is not possible: in all cases the investor is able to recover her initial investment. Thus, the risk measured by variance or standard deviation measures uncertainty associated with random but positive returns. Out of these portfolios the BPT investor chooses the one with the highest return. At the same time, an investor following Markowitz approach chooses the portfolio with the a priori fixed standard deviation. By reducing the level of risk this investor denies herself the chance of getting very high returns by ensuring that she cannot lose money. This is the underlying idea behind the criticism of symmetric risk measures such as variance and standard deviation.

6. Conclusion

Our results show the Markowitz (1952) portfolio selection model can be a viable and cost-effective tool for investors despite the fact that the symmetry of risk with respect to the expectation is non-intuitive. We used historical stock price observations with 71 assets over 1535 days to run portfolio simulations of two different kinds investors. One investor in our simulations used symmetric risk measure and chose her optimal portfolio by mean-variance analysis. The other one considered downside risk measure following the BPT. We obtained that the portfolio choices of the two are very similar.
Our results contrast assertions of a number of important papers that claim that investors perceive risk to be the downside deviations from the objective levels of returns rather than any deviations. Our research into security constraints and security set offers unique empirical evidence that despite the intuitive clumsiness of the symmetric risk measures the mean-variance framework produces outcomes that coincide with the alternative portfolio theories based on safety-first principle and downside risk. The value of this study rests in part with the fact that empirical studies on downside vs. symmetric risk are very limited.

A promising next step would be to consider new data in light of the ongoing financial crisis and credit crunch. As investors become more prudent with lack of attractive opportunities perhaps behavioral portfolio theory become more likely to hold.
References


Table 1. Descriptive Statistics

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<th>StDev*10^{-3}</th>
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<td>Average</td>
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<td>0.05</td>
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Table 2. Diversification Effects

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<th>Number of Assets</th>
<th>Reduction of Variance*</th>
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<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0.35</td>
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*proportion by which the variance is reduced

Table 3. Results

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<td>117</td>
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<td>120</td>
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<td>82</td>
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<td>$N_S/140$</td>
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<td>1</td>
<td>0</td>
<td>0</td>
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<td>7</td>
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</tbody>
</table>

Total
Figure 1

Optimal Portfolios

State

Payoff

Figure 2. Diversification Effect

Variance

Number of Assets in The Portfolio

Shefrin and Statman
Markowitz
Figure 3

Set of all 100,000 portfolios
Set of all 100,000 portfolios (blue)
Security set $r^* = 0$ and $\alpha=0.3$ (green)
Set of all 100,000 portfolios (blue)

Security set $r^* = 0$ and $\alpha = 0.3$ (green)

Security set $r^* = 0$ and $\alpha = 0.2$ (red)

Security set $r^* = 0$ and $\alpha = 0.1$ (light blue)

Security set $r^* = 0$ and $\alpha = 0$ (purple/pink)
Set of all 100,000 portfolios (blue)

Security set $r^* = 0$ and $\alpha = 0.3$ (green)

Security set $r^* = 0.05$ and $\alpha = 0.3$ (en rouge)

Security set $r^* = 0.1$ and $\alpha = 0.3$ (light blue)
<table>
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<tr>
<th>D.R. n° 1</th>
<th>&quot;Bertrand Oligopoly with decreasing returns to scale&quot;, J. Thépot, décembre 1993</th>
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<td>&quot;Bounded Rationality in Microeconomics&quot;, J. Thépot, mai 1993</td>
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<td>D.R. n° 7</td>
<td>&quot;Vendre ou louer ; un apport de la théorie des jeux&quot;, J. Thépot, avril 1994</td>
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<td>D.R. n° 9</td>
<td>&quot;Les actions à réinvestissement optionnel du dividende&quot;, C. Marie-Jeanne - P. Roger, janvier 1995</td>
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<td>D.R. n° 10</td>
<td>&quot;Forme optimale des contrats d'assurance en présence de coûts administratifs pour l'assureur&quot;, S. Spaeter, février 1995</td>
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<td>D.R. n° 11</td>
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<td>D.R. n° 20</td>
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