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How harmful are cuts in public employment and wage in times of high unemployment?

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Abstract

Since 2010 public employment and public-sector salaries have been significantly reduced in most Euro Area member states. In this article we show to what extent these cuts in the public sector have been costly particularly in terms of employment and output. In a New Keynesian model with a two-sector labor market, we demonstrate that the cost of these spending cuts on employment and output is significantly larger in periods of high unemployment. We also exhibit that cuts in public employment and wage in a Eurozone prone to high unemployment have only a limited ability to reduce deficit.

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1 Introduction

Eight years after the crisis, European economies seem to be enmeshed in a period of weak GDP growth. The European Central Bank forecasts a 1.4% growth of GDP for 2015, that is significantly lower than in the US and in the UK (respectively 2.4% and 2.1%). Also, despite positive elements like a falling Euro and historically low oil prices, unemployment falls only slightly and remains above 10% in the Euro Area. This weak performance of the European economy could be due, at least partly, to the fiscal orientation chosen by most Euro Area members in recent years: since 2010, European policymakers have implemented particularly large fiscal consolidation plans.

Blanchard and Leigh (2013) show that forecasters have underestimated the cost on GDP growth of recent fiscal consolidation episodes leading to large growth forecast errors. In other words, fiscal austerity in the Euro Area would have had particularly strong and unexpected negative effects on GDP since fiscal multipliers would have been large during this period. For instance, a 5% contraction of GDP was forecasted in Latvia while the actual contraction was 18%. In Hungary, the forecasted contraction was 1% but the actual figure was 6.7%.

Different reasons have been advanced to explain unusually large fiscal multipliers. As summarized by Blanchard and Leigh (2013), at least three factors can be pointed out: central banks' interest rates close to 0 (the Zero Lower Bound), badly functioning financial markets and a large fall in GDP following the crisis. Regarding the latter element, there would be a sizable difference in the output fiscal multiplier according to the position of the economy over the business cycle. This result has been highlighted in recent empirical studies and notably in Auerbach and Gorodnichenko (2012): estimates based on US data indicate that the output fiscal multiplier is close to 0 in normal times but can reach 2.5 in periods of recession.[1]

Alongside these empirical contributions, only few papers investigate the transmission channels at work in a theoretical framework. Sims and Wolff (2013) examine in a small-scale DSGE model the size of the fiscal multiplier

[1]Other studies bring similar results. See among others Creel et al. (2011), Baum et al. (2012) or Batini et al. (2012)

along the business cycle. The authors show that a standard (non-linear) New Keynesian model can generate a countercyclical output fiscal multiplier. The intuition is straightforward: marginal utility of consumption is larger during economic downturn because of a lower average consumption level. In a Ricardian economy, households reduce less their level of consumption following the rise in real interest rates after fiscal expansions during economic downturn, *i.e* when the marginal utility of consumption is high. A lower negative wealth effect of public consumption on private consumption thus implies a larger output fiscal multiplier in bad times. As detailed later on, our article is closely related to Sims and Wolff (2013) since a larger output fiscal multiplier in bad times is also obtained through a lower crowding-out effect of government expenditure on private activity.

In the present article, we focus on the non-linear effects of cuts in public employment and public wage on total employment and economic activity. These two fiscal instruments have been extensively used in the recent European austerity plans. In Spain, 14000 public jobs have been cut and a replacement rate of 10% has been implemented for the period 2012-2013. Also, public-sector salaries were decreased by 5% in 2010 and then frozen in 2011. In Greece, fiscal efforts have been particularly violent: only 10% of retirements have been replaced while public-sector salaries have been frozen. Moreover, thirteenth and fourteenth month pay have been removed. Overall, compensations to employees in the public sector are frozen in Germany, France, Italy, Greece and Portugal and public employment is significantly reduced.

In Creel et al. (2011), the authors estimate a significant difference as to the effects of changes in public employment on output according to the position of the economy over the business cycle. In the long run, the output fiscal multiplier is estimated to 1.5 in good times and to -1.1 during economic downturn. Michaillat (2014) analyses the non-linear effects of public employment on private employment. In a search and matching model for the labor market in which both a public and a private sector coexist, a rise in public vacancies tends to crowd-out private employment despite that the effects on total employment remains positive.[2] Michaillat (2014) studies the

[2]Ramey (2012) estimates the effects of public employment on private activity and also argue for a negative effect of public employment on private employment

effects of a rise in public vacancies on private and total employment according to the unemployment level at the steady state. The main result is that the higher the unemployment rate, the lower the crowding-out effect of public employment on the private sector. This induces that decreasing the number of public employees is more costly in terms of total employment when the unemployment rate is large.

In the present paper, we also demonstrate that cuts in public employment and public wage affect more negatively total employment when the unemployment rate is large. In addition, we exhibit that cuts in public employment and public-sector salaries trigger a larger degradation of output in periods of high unemployment. As a consequence, the reduction in debt induced by restrictive fiscal policies is significantly lower when cuts in government employment and public-sector salaries are implemented in times of high unemployment. Hence, this paper argues that recent fiscal contractions based on public employment have been particularly harmful in the Euro Area.

We construct a large-scale DSGE model with a two-sector labor market *à la* Mortensen and Pissarides. The model is solved at the second order to take into account the influence of the steady-state unemployment on the response of the economy following cuts in government employment and wage.[3] As in Michaillat (2014), we show that cuts in government employment and public-sector salaries are more harmful in terms of employment when the initial unemployment rate is high. Let us consider the case of a rise in public vacancies. Intuitively, the larger the number of job seekers, the lower the crowding-out effect of government employment on private employment. Hence, cuts in government employment have more costly effects on total employment when unemployment is already high.

Unlike Michaillat (2014) we also focus on the response of output following fiscal policy shocks. We demonstrate that the greater negative effect on employment of cuts in public employment and wage in times of high unemployment triggers a larger negative effect on output. The stronger decrease in employment tends to generate a larger degradation of consumption of *hand-*

[3]Hairault et al. (2010), analyzing the welfare costs of business cycles thanks to the matching unemployment theory show that the search and matching framework can be quite non-linear.

to- mouth households. Moreover, the private-sector real wage remains larger when cuts in public-sector employment and salaries are implemented in times of high unemployment. As a consequence, inflation and then the real interest rate tend to be larger in the case of a high steady-state unemployment rate so that the response of Ricardian consumption is better in this case.

This article is close to Sims and Wolff (2013) since we focus on state-dependent output fiscal multipliers according to the position over the business cycle in a large-scale DSGE model and our result is based on a lower crowding-out effect of fiscal policy on private consumption. However, we depart from Sims and Wolff (2013) since a lower crowding-out effect on private consumption is, in our case, not due to a different behavior from the representative household according to the position over the business cycle but to larger inflation pressures in bad times following a negative fiscal shock.

Sims and Wolff (2013) argue for a larger output fiscal multiplier during economic downturn due to a larger marginal utility of consumption. It is important to note that our model does not include this transmission channel. On the contrary, our definition of the steady-state value of Ricardian's consumption implies a lower marginal utility of consumption during economic downturn. The transmission channel we highlight in this paper is not in contradiction with the explanation found in Sims and Wolff (2013). Especially, the coexistence of these two effects could partly explain the sizeable difference found in the literature about the size of the output fiscal multiplier according to the position of the economy over the business cycle.

The rest of the paper is organized as follows: section 2 presents the model, section 3 describes the calibration and the simulation strategy. Section 4 highlights the main results of the paper and section 5 concludes.

2 The DSGE model

The model used in this paper features nominal rigidity on prices and matching frictions on the labor market in which both a public and a private sector are introduced. We introduce an efficient Nash wage bargaining in which the public wage directly affects the determination of the private wage and thus employment in both sectors.

2.1 Definitions and the matching process

Let us first define the non-employed pool $1-(1-\rho)E_t^{tot}$ such as:

$$1-(1-\rho)E_t^{tot} = U_t + \rho E_t^{tot}, \tag{2.1}$$

where E_t^{tot} denotes the employed workers and U_t the pool of unemployed workers. The destruction rate ρ is assumed to be exogenous.

Moreover, the pool of job seekers S_t is expressed as

$$S_t = U_t + \rho E_t^{tot}. \tag{2.2}$$

Also, in the spirit of Trigari (2006), assuming that a new job becomes productive only in the following period and assuming that a match can be instantaneously broken, employment in a particular sector E_t^i can be expressed as:

$$E_t^i = (1-\rho)E_{t-1}^i + p_{t-1}^i(1-\rho)S_{t-1}, \tag{2.3}$$

with $i = p, g$ where p characterizes the private sector and g the public sector. The job-finding probability in the sector i, p_t^i, is defined later on. With these definitions, it is important to note that total employment is a predetermined variable.

Finally, the dynamic of job seekers is given by

$$S_t = (1-p_{t-1}^p - p_{t-1}^g)S_{t-1} + \rho(p_{t-1}^p + p_{t-1}^g)S_{t-1} + \rho(E_{t-1}^p + E_{t-1}^g). \tag{2.4}$$

According to equation (2.4), the number of job seekers in the current period is equal to the number of job seekers who did not find a job neither in the private sector nor in the public sector in the previous period plus the number of jobs which are destroyed in the previous period. Finally, we assume that there is a trial period: a worker can match a firm in the beginning of the period but the relationship can be broken at the end of the period exogenously.

Let us now define the matching process M_t^i that occurs on a specific labor market sector, such as:

$$M_t^i = \kappa_e^i (S_t)^{\varphi^i} (V_t^i)^{(1-\varphi^i)}, \tag{2.5}$$

where κ_e^i denotes the matching technology in a particular sector while φ^i denotes the elasticity of employment for a supplementary unemployed worker. V_t^i defines the number of vacancies in the sector i. Vacancies in the public sector are assumed to be set as exogenous by the government.

We can therefore set the following usual definitions:

$$p_t^i = \frac{M_t^i}{S_t}, \tag{2.6}$$

$$\text{and} \quad q_t^i = \frac{M_t^i}{V_t^i} \tag{2.7}$$

with p_t^i the job finding probability in the sector i and q_t^i the probability for a firm to fill a vacancy.

The labor market tightness (LMT thereafter) can be defined as:

$$\theta_t^i = \frac{V_t^i}{S_t} = \frac{p_t^i}{q_t^i}. \tag{2.8}$$

2.2 Households' decisions

In this model two different types of agents are introduced. We assume a share μ of non-Ricardian (*hand-to-mouth*) households and a share $(1-\mu)$ of Ricardian households. The difference between both types of households is their ability to participate in financial markets. *Hand-to-mouth* consumers can neither loan nor save so that they simply consume their disposable income in each period, while Ricardian households can hold a riskless asset that allows them to optimize their consumption inter-temporally. Also, Ricardian households invest in physical capital that they then loan to firms. Both types of households formulate similar labor market decisions.

2.2.1 Ricardian households

As in Merz (1995), we consider a representative Ricardian household who maximizes its lifetime utility, with instantaneous utility defined as:

$$u(C_t^o, C_{t-1}^o, G_t, e_{it}) = \frac{(C_t^o - HC_{t-1}^o)^{1-\sigma_c} - 1}{1-\sigma_c} + M^o(e_{jt}) \tag{2.9}$$

where C_t^o denotes consumption of Ricardian households. Additively separable preferences for consumption and labor are introduced in an usual manner with σ_c representing the inter-temporal elasticity of substitution of consumption. The consumption decision is subject to habit formation H. The function $M^o(e_{jt})$ defines the amount of leisure in terms of utility with regard to the status of the household on the labor market.

Following Ravn (2005, 2008), e_{jt} with $j = n, u, l$ denotes the level of leisure according to the status of the household on the labor market *i.e.* e_{nt} for an employed worker, e_{ut} for an unemployed worker and e_{lt} for an inactive household such as:

$$e_{nt} = 1 - h - s, \tag{2.10}$$

$$e_{ut} = 1 - s, \tag{2.11}$$

$$e_{lt} = 1, \tag{2.12}$$

where h denotes hours worked that we assume as exogenous and s denotes a fixed cost to participate in the labor market.

Function $M^o(e_{it})$ contains the different possible statuses of a worker on the labor market, such as:

$$M^o(e_{it}) = \frac{[(E_t^{op} + E_t^{og})(1-h-s)^{1-\zeta} + S_t^o(1-s)^{1-\zeta} + (1 - (E_t^{op} + E_t^{og}) - S_t^o)]}{1-\zeta} \tag{2.13}$$

where $-1/\zeta$ is the Frisch elasticity of labor supply and S_t^o denotes the job seekers among Ricardian households. E_t^{op} denotes employment of Ricardian households in the private sector while E_t^{og} denotes employment of Ricardian households in the public sector.

The optimization problem for the representative Ricardian household is expressed as:

$$\max_{C_t^o, K_t^o, B_t, E_t^o, S_t^o, I_t^o} E_t \sum_{s=t}^{\infty} \beta^s u(C_{t+s}^o, C_{t-1+s}^o, G_{t+s}, e_{t+s}). \tag{2.14}$$

subject to

$$\begin{aligned} (1+\tau_t^c)C_t^o + \frac{B_t}{P_t} + I_t^o \leq R_{t-1}^k K_{t-1} + \frac{R_{t-1}B_{t-1}}{P_t} + b(S_t^o) \\ +(1-\tau_t^w)[W_t^g h E_t^{og} + W_t^p h E_t^{op}] \quad (2.15) \\ K_t^o = (1-\delta^k)K_{t-1}^o + [1 - A(I_t^o/I_{t-1}^o)]I_t^o \quad (2.16) \\ E_t^{op} = (1-\rho)E_{t-1}^{op} + p_{t-1}^p(1-\rho)S_{t-1}^o \quad (2.17) \\ E_t^{og} = (1-\rho)E_{t-1}^{og} + p_{t-1}^g(1-\rho)S_{t-1}^o \quad (2.18) \\ S_t^o = (1-p_{t-1}^p - p_{t-1}^g)S_{t-1}^o + \rho(p_{t-1}^p + p_{t-1}^g)S_{t-1}^o + \rho(E_{t-1}^{op} + E_{t-1}^{og}) \quad (2.19) \end{aligned}$$

Equation (2.14) can be reduced to the following Bellman equation:

$$\begin{aligned} \Omega_t^o(K_t^o, E_t^o, B_t, I_t^o) = \max_{C_t^o, K_t^o, S_t^o, n_t^o, B_t, I_t} \Bigg\{ \frac{(C_t^o - HC_{t-1}^o)^{1-\sigma_c}}{1-\sigma_c} + \frac{\zeta_g g_t^{1-\sigma_c} - 1}{1-\sigma_c} \\ + \frac{[(E_t^{op} + E_t^{og})(1-h-s)^{1-\zeta} + S_t^o(1-s)^{1-\zeta} + (1 - (E_t^{op} + E_t^{og}) - S_t^o)]}{1-\zeta} \Bigg\} \\ + \beta\Omega_{t+1}^o(K_{t+1}^o, E_{t+1}^o, B_t, I_t^o), \end{aligned} \tag{2.20}$$

where β is the discount factor. Equation (2.15) represents the household's budget constraint. Households have access to a riskless asset B_t. Furthermore, households invest I_t^o in physical capital K_t^o and loan it to the firms at a rate R_t^k. δ^k defines the depreciation rate of capital, R_t the nominal interest rate equals to $\frac{1}{\beta}$ at the steady state and b the unemployment benefits. W_t^g and W_t^p are the real wages respectively in the public and in the private sector. P_t defines the consumer price index (CPI thereafter). τ_t^c represents a VAT and τ_t^w a labor income tax . Equation (2.16) represents the law of motion of capital accumulation. We introduce an adjustment cost to investment changes with $A(I_t^o/I_{t-1}^o) = \frac{\kappa}{2}(I_t^o/I_{t-1}^o - 1)^2$ in lines with Christiano et al. (2005) or Smets and Wouters (2007), with κ a constant cost associated to investment decisions.

First order conditions with respect to respectively C_t^o, B_t, I_t^o, K_t^o, E_t^{op}, E_t^{og} and S_t^o yield:

$$\lambda_t^{rio} = \frac{[C_t^o - HC_{t-1}^o]^{-\sigma_c} - \beta H E_t\{[C_{t+1}^o - HC_t^o]^{-\sigma_c}\}}{1+\tau_t^c} \tag{2.21}$$

$$\lambda_t^{rio} = r_t \beta E_t \left[\frac{\lambda_{t+1}^{rio}}{\pi_{t+1}}\right], \tag{2.22}$$

$$1 = Q_t[1 - A(I_t/I_{t-1})] \tag{2.23}$$

$$Q_t = \beta E_t \left[\frac{\lambda_{t+1}^{rio}}{\lambda_t^{rio}}[(1-\delta^k)Q_{t+1} + R_t^k]\right] \tag{2.24}$$

$$\begin{aligned}\lambda_t^{E_{op}} = (1-\tau_t^w)\lambda_t^{rio} W_t^p h - \frac{1-(1-h-s)^{1-\zeta}}{1-\zeta} \\ +\beta E_t[(1-\rho)(\lambda_{t+1}^{E_{op}} - \lambda_{t+1}^{S_o}) + \lambda_{t+1}^{S_o}]\end{aligned} \tag{2.25}$$

$$\begin{aligned}\lambda_t^{E_{og}} = (1-\tau_t^w)\lambda_t^{rio} W_t^g h - \frac{1-(1-h-s)^{1-\zeta}}{1-\zeta} \\ +\beta E_t[(1-\rho)(\lambda_{t+1}^{E_{og}} - \lambda_{t+1}^{S_o}) + \lambda_{t+1}^{S_o}]\end{aligned} \tag{2.26}$$

$$\begin{aligned}\lambda_t^{S_o} = b\lambda_t^{rio} - \frac{1-(1-s)^{1-\zeta}}{1-\zeta} \\ +(1-p_t^p-p_t^g)\beta E_t[\lambda_{t+1}^{S_o}] + \rho(p_t^p+p_t^g)\beta E_t[\lambda_{t+1}^{S_o}] \\ +(1-\rho)\beta E_t[p_t^p \lambda_{t+1}^{E_{op}} + p_t^g \lambda_{t+1}^{E_{og}}]\end{aligned} \tag{2.27}$$

where $\pi_{t+1} = p_{t+1}/p_t$ defines the CPI inflation rate, λ_t^{rio} the marginal utility of consumption for Ricardians, $\lambda_t^{E_{op}}$ the marginal utility of working in the private sector, $\lambda_t^{E_{op}}$ the marginal utility of working in the public sector and $\lambda_t^{S_o}$ the marginal utility to be currently a job seeker.

Equation (2.25) defines the value of a job for a Ricardian household in the private sector while equation (2.26) determines the value of a job in the public sector. Also, equation (2.27) describes the decision for a Ricardian worker to participate in the labor market.

2.2.2 ***Hand-to-mouth*** consumers

Non-Ricardian households do not maximize consumption inter-temporally and simply consume their disposable income each period. For a representative non-Ricardian household, consumption can be expressed as:

$$(1+\tau_t^c)C_t^r = (1-\tau_t^w)[W_t^g hE_t^g + W_t^p hE_t^p] + bS_t^r \tag{2.28}$$

with C_t^r the consumption of non-Ricardians households.

Similarly to Ricardian households, the utility function of *hand-to-mouth* households is given by:

$$u(C_t^r, C_{t-1}^r, G_t, e_{it}) = \frac{(C_t^r - HC_{t-1}^r)^{1-\sigma_c} - 1}{1-\sigma_c} + \frac{\zeta_g G_t^{1-\sigma_c}}{1-\sigma_c} + M^r(e_{jt}) \tag{2.29}$$

with

$$M^r(e_{it}) = \frac{[(E_t^{rp} + E_t^{rg})(1-h-s)^{1-\zeta} + S_t^r(1-s)^{1-\zeta} + (1-(E_t^{op} + E_t^{rg}) - S_t^r)]}{1-\zeta} \tag{2.30}$$

The corresponding Bellmann equation and constraints for this optimization program are therefore:

$$\Omega_t^r = \max_{S_t^r, E_t^r, E_t^g} \left\{ \frac{(C_t^r - HC_{t-1})^{1-\sigma_c}}{1-\sigma_c} + \frac{[(E_t^{rp} + E_t^{rg})(1-h-s)^{1-\zeta} + S_t^r(1-s)^{1-\zeta} + (1-(E_t^{rp} + E_t^{rg}) - S_t^r))]}{1-\zeta} \right\} + \beta\Omega_{t+1}^r \tag{2.31}$$

s.t.

$$(1+\tau_t^c)C_t^r \leq (1-\tau_t^w)[W_t^g hE_t^{rg} + W_t^p hE_t^{rp}] + bS_t^r \tag{2.32}$$

$$E_t^{rp} = (1-\rho)E_{t-1}^{rp} + p_{t-1}^p(1-\rho)S_{t-1}^r \tag{2.33}$$

$$E_t^{rg} = (1-\rho)E_{t-1}^{rg} + p_{t-1}^g(1-\rho)S_{t-1}^r \tag{2.34}$$

$$S_t^r = (1-p_{t-1}^p - p_{t-1}^g)S_{t-1}^r + \rho(p_{t-1}^p + p_{t-1}^g)S_{t-1}^r + \rho(E_{t-1}^{rp} + E_{t-1}^{rg}) \tag{2.35}$$

First order conditions with respect to E_t^{rp}, E_t^{rg} and S_t yield:

$$\lambda_t^{E_{rp}} = (1-\tau_t^w)\lambda_t^{rir} W_t^p h - \frac{1-(1-h-s)^{1-\zeta}}{1-\zeta} + \beta E_t[(1-\rho)(\lambda_{t+1}^{E_{rp}} - \lambda_{t+1}^{S_r}) + \lambda_{t+1}^{S_r}] \tag{2.36}$$

$$\lambda_t^{E_{rg}} = (1-\tau_t^w)\lambda_t^{rir} W_t^g h - \frac{1-(1-h-s)^{1-\zeta}}{1-\zeta} + \beta E_t[(1-\rho)(\lambda_{t+1}^{E_{rg}} - \lambda_{t+1}^{S_r}) + \lambda_{t+1}^{S_r}] \tag{2.37}$$

$$\lambda_t^{S_r} = b\lambda_t^{rir} - \frac{1-(1-s)^{1-\zeta}}{1-\zeta} + (1-p_t^p-p_t^g)\beta E_t[\lambda_{t+1}^{S_r}] + \rho(p_t^p+p_t^g)\beta E_t[\lambda_{t+1}^{S_r}] + (1-\rho)\beta E_t[p_t^p\lambda_{t+1}^{E_{rp}} + p_t^g\lambda_{t+1}^{E_{rg}}] \tag{2.38}$$

where $\lambda_t^{E_{rp}}$ is the marginal utility of working in the private sector for a non-Ricardian household, respectively $\lambda_t^{E_{rg}}$ in the public sector and $\lambda_t^{S_r}$ denotes the marginal utility for a non-Ricardian household to seek employment on the labor market.

Equation (2.36) defines the value of a job in the private sector for a non-Ricardian household while (2.37) defines the value of a job in the public sector. Also, equation (2.38) relates to the decision of a non-Ricardian worker to seek a job.

Maximization of (2.31) with respect to C_t^r yields the marginal utility of consumption for non-Ricardian households, such as:

$$\lambda_t^{rir} = \frac{(C_t^r - HC_{t-1}^r)^{\sigma_c} - \beta E_t[H(C_{t+1}^r - HC_t^r)^{\sigma_c}]}{1+\tau_t^c} \tag{2.39}$$

2.3 Firms

For the purposes of the model, we need to introduce three kinds of firms as in Trigari (2006). First, some firms we refer as "producers" produce goods with labor and private capital in a competitive environment. The producers then sell their aggregate goods to "intermediate firms", which transform the aggregate good on a continuum of differentiated goods in a monopolistic competition environment. The intermediate firms are the price-setters and set their optimal price subject to nominal rigidity as in Calvo (1983). Finally, a continuum of "final goods firms" in a competitive environment purchase the differentiated intermediate goods and package them to sell it to consumers. This dissociation between producers and intermediate firms is necessary because introducing price-setting at the producer level would greatly complicate the decision of these firms on the labor market. However, this simplifying assumption has no important consequences neither on the price dynamic nor on the labor market dynamics. [4]

2.3.1 Producers

A representative firm in a perfectly competitive environment seeks to maximize its profits according to the following optimization program:

$$\max_{\tilde{K}_t, E_t^p, V_t} E_0 \sum_{t=0}^{\infty} \beta_{t,t+1} \{Y_t - R_t^k \tilde{K}_t - W_t^p E_t^p h - \kappa^v V_t\} \tag{2.40}$$

s.t.

$$Y_t = \epsilon_t^A [\tilde{K}_t]^\alpha [E_t^p h]^{1-\alpha} \tag{2.41}$$

$$E_t^p = (1-\rho) E_{t-1}^p + q_{t-1}^p V_{t-1}^p \tag{2.42}$$

where $\beta_{t,t+1} = \beta \dfrac{\lambda_{t+1}^{rio}}{\lambda_t^{rio}}$ defines the firm's discount factor. Moreover, the producer takes the probability to fill a vacancy q_t^p as given. V_t^p denotes the vacancies posted by the producer and κ^v an unitary cost. The accumulated capital is assumed to be used by firms with a lag, such as $\tilde{K}_t = K_{t-1}$. ϵ_t^A denotes a Total-Factor Productivity (TFP thereafter) shock and follows an AR(1) process such as:

$$\left(\frac{\epsilon_t^A}{\epsilon_s^A}\right) = \left(\frac{\epsilon_{t-1}^A}{\epsilon_s^A}\right)^{\rho_\epsilon} exp(\varepsilon_t^a),$$

[4] For more details, Christoffel et al. (2009a) made a survey on the implication of this assumption. In the spirit of Kuester (2010), Sveen and Weinke (2007) and Thomas (2011), Christoffel et al. (2009b) demonstrate that the dissociation assumption not only has no spurious consequences but also helps the standard Keynesian model to match stylized facts regarding the response of inflation to monetary shocks.

where ϵ_s^A stands for the TFP at the steady-state. $exp(\varepsilon_t^a)$ is an *iid* exogenous disturbance and ρ_ϵ the duration of the shock.

The problem (2.40) can be represented as a Bellman equation such as:

$$V(\Omega_t) = \max_{k_t, E_t^p, V_t} \{Y_t - R_t^k k_t - W_t^p E_t^p h - \kappa^v V_t + \beta \frac{\lambda_{t+1}^{rio}}{\lambda_t^{rio}} V(\Omega_{t+1})\} \tag{2.43}$$

Under the free entry condition, the first order conditions with respect to vacancy posting and employment yield:

$$\frac{\kappa^v}{q_t^p} = \beta_{t,t+1} \frac{\lambda_{t+1}^{rio}}{\lambda_t^{rio}} \lambda_{t+1}^{E_f} \tag{2.44}$$

$$\lambda_t^{E_f} = (1-\alpha)\frac{Y_t}{E_t^p} - W_t^p h + (1-\rho)\beta_{t,t+1}\frac{\lambda_{t+1}^{rio}}{\lambda_t^{rio}} \lambda_{t+1}^{E_f} \tag{2.45}$$

Equation (2.44) defines the value of a posted vacancy and (2.45) the value of a job for a producer.

Cost minimization subjects to equation (2.41) implies the following first order conditions:

$$R_t^k = \frac{\alpha Y_t}{K_t} mc_t \tag{2.46}$$

$$x_t = (1-\alpha) mc_t \frac{Y_t}{E_t^p h} - W_t^p h, \tag{2.47}$$

where mc_t is the firms' marginal cost. Equation (2.46) characterizes the demand of capital by the producers and equation (2.47) defines the marginal cost of labor x_t.

2.3.2 Intermediate firms, final goods firms and Calvo price-setting

There is a continuum j over $[0;1]$ of intermediate firms that purchase the homogeneous goods from the producers at their marginal cost. Intermediate firms then transform the homogeneous goods on a continuum j of goods and sell them at the final goods firms.

Final goods firms produce a package of the intermediate differentiated goods according to:

$$Y_t = \left[\int_0^1 Y_{jt}^{\frac{\varepsilon - 1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon - 1}}, \tag{2.48}$$

where ε is the elasticity of substitution across intermediate goods. Demand for each intermediate good is of the form:

$$Y_{jt} = \left(\frac{P_{jt}}{P_t} \right)^{-\varepsilon} Y_t, \tag{2.49}$$

with the following definition for the consumer price index P_t:

$$P_t = \left[\int_0^1 P_{jt}^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}}, \tag{2.50}$$

where P_{jt} defines the price of good j in the period t.

Following Calvo (1983), intermediate firms are allowed to re-optimize their price only with a probability $\theta_p \in [0, 1]$ each period. This probability is assumed to be independent from the re-optimization decision taken in the last period.

An intermediate firm re-optimizes its price at period t seek to maximize its profit such as:

$$E_t \sum_{k=0}^{\infty} (\beta\theta_p)^s \frac{\lambda_{t+s}^{rio}}{\lambda_t^{rio}} \left[\frac{P_{j,t}}{P_{t+s}} - mc_{t+s} \right] Y_{j,t+s}, \tag{2.51}$$

subject to the demand function expressed in equation (2.49). The first order condition yields:

$$P_{jt}^* = \frac{\varepsilon}{\varepsilon - 1} \frac{E_t \sum_{s=0}^{\infty} (\beta\theta_p)^s \frac{\lambda_{t+s}^{rio}}{\lambda_t^{rio}} [mc_{t+s} P_{t+s}^{\varepsilon} Y_{t+s}]}{E_t \sum_{s=0}^{\infty} (\beta\theta_p)^s \frac{\lambda_{t+s}^{rio}}{\lambda_t^{rio}} [P_{t+s}^{\varepsilon-1} Y_{t+s}]} \tag{2.52}$$

where P_{jt}^* is the optimal price of the intermediate firm j and $\frac{\varepsilon}{\varepsilon - 1}$ the desired (natural) mark-up. Finally, the law of motion for aggregate prices is given by

$$P_t = [(1-\theta_p)P_t^{*1-\varepsilon} + \theta_p P_{t-1}^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}}. \tag{2.53}$$

Combination of equations (2.52) and (2.53) yield the New-Keynesian Phillips Curve.

2.4 Wage bargaining

The union utility corresponds to the mean of the surplus on employment of all its members. With μ beeing the share of non-Ricardian households, the union utility Υ_t can be expressed as:

$$\Upsilon_t = (1-\mu)[\lambda_t^{E_{op}} - \lambda_t^{S_o}] + \mu[\lambda_t^{E_{rp}} - \lambda_t^{S_r}] \tag{2.54}$$

The surplus for a Ricardian household to stay employed following the wage bargaining is given by:

$$\begin{aligned}\lambda_t^{E_{op}} - \lambda_t^{S_o} = (1-\tau_t^w)\lambda_t^{rio}W_t^p h - \lambda_t^{rio} b + \frac{(1-h-s)^{1-\zeta} - (1-s)^{1-\zeta}}{1-\zeta} \\ +\beta E_t[(1-p_t)(1-\rho)(\lambda_{t+1}^{E_{op}} - \lambda_{t+1}^{S_o}) - p_t^g(1-\rho)(\lambda_{t+1}^{E_{og}} - \lambda_{t+1}^{S_o})]\end{aligned} \tag{2.55}$$

and similarly for the non-Ricardian workers:

$$\begin{aligned}\lambda_t^{E_{rp}} - \lambda_t^{S_r} = (1-\tau_t^w)\lambda_t^{rir}W_t^p h - \lambda_t^{rir} b + \frac{(1-h-s)^{1-\zeta} - (1-s)^{1-\zeta}}{1-\zeta} \\ +\beta E_t[(1-p_t)(1-\rho)(\lambda_{t+1}^{E_{rp}} - \lambda_{t+1}^{S_o}) - p_t^g(1-\rho)(\lambda_{t+1}^{E_{rg}} - \lambda_{t+1}^{S_r})]\end{aligned} \tag{2.56}$$

2.4.1 Nash product and efficient bargaining

Under the free entry condition, the Nash product can be expressed as:

$$\mathcal{N}_t = \Upsilon_t^{\eta}[\lambda_t^{E_f}]^{1-\eta}, \tag{2.57}$$

where η denotes the union bargaining power.

In the case of efficient bargaining, firms and union jointly determine the real wage but not the hours worked since we assume them as exogenous.

Maximization of the Nash product subject to the private real wage leads to the following optimal rule for the surplus allocation:

$$\eta \frac{\partial \Upsilon_t}{\partial W_t^p} \lambda_t^{E_f} = (1-\eta) \frac{-\partial \lambda_t^{E_f}}{\partial W_t^p} \Upsilon_t \tag{2.58}$$

After several calculation steps (fully described in appendix B), we obtain this rule for the private real wage (net of the income tax):

$$\begin{aligned}(1-\tau_t^w)W_t^p h = {} & \eta(1-\alpha)(1-\tau_t^w)\frac{Y_t}{E_t^p} + (1-\eta)\left[b + \frac{(1-s)^{1-\zeta} - (1-h-s)^{1-\zeta}}{(1-\zeta)(\mu\lambda_t^{rir} + (1-\mu)\lambda_t^{rio}}) \right] \\ & + \eta(1-\rho)E_t\left\{\beta_{t,t+1}\left[1-(1-p_t^p)(1-\tau_{t+1}^w)\tilde{\Lambda}_{t+1}\right]\lambda_{t+1}^{E_f}\right\} \\ & + (1-\eta)(1-\rho)p_t^g \beta E_t[\Lambda_t(\lambda_{t+1}^{E_{rg}} - \lambda_{t+1}^{S_r}) + (1-\Lambda_t)(\lambda_{t+1}^{E_{og}} - \lambda_{t+1}^{S_o})],\end{aligned} \tag{2.59}$$

with $\Lambda_t = \dfrac{\mu\lambda_t^{rir}}{\mu\lambda_t^{rir} + (1-\mu)\lambda_t^{rio}}$ the relative part of non-Ricardian consumers in the consumer pool and $\tilde{\Lambda}_t = \dfrac{\mu\lambda_t^{rir} + (1-\mu)\lambda_t^{rio}}{\mu\lambda_{t-1}^{rir} + (1-\mu)\lambda_{t-1}^{rio}}$.

2.5 Monetary and fiscal policies

Each period, the monetary authority sets the nominal interest rate according to the following standard Taylor rule:

$$\frac{R_t}{R_s} = \left(\frac{R_{t-1}}{R_s}\right)^{\alpha^r} \left(\frac{Y_t}{Y_s}\right)^{\alpha^y} \left(\frac{\pi_t}{\pi_s}\right)^{\alpha^\pi} \tag{2.60}$$

with R_s, Y_s and π_s the nominal interest rate, output and inflation at the steady state, respectively. α^r is the degree of inertia of the nominal interest rate and α^y and α^π the relative weights given by the monetary authority to the stabilization of output and inflation.

Each period, the budget constraint each period for the government is given by:

$$\frac{B_{t+1}}{r_t} - B_t = C^g + bS_t W_t^g E_t^g h - [\tau_t^c C_t + \tau_t^w (W_t^p E_t^p h)] \tag{2.61}$$

The debt in GDP share Ψ_t is given by:

$$\Psi_t = \frac{B_t}{Y_t P_t}. \tag{2.62}$$

We assume that VAT responds to public debt according to the following rule:

$$\frac{\tau_t^c}{\tau_s^c} = \left(\frac{\tau_{t-1}^c}{\tau_s^c}\right)^{\rho^{\tau_t^c}} \left(\frac{\Psi_t}{\Psi_s}\right)^{\rho^{\Psi}}, \tag{2.63}$$

where $\alpha^{\tau_t^c}$ is the constant VAT AR(1) coefficient and where α^B denotes the the degree of reaction of VAT to a variation of public debt. τ_s^c and Ψ_s denote the level of VAT and of public debt in GDP share at the steady state, respectively.

Public wage and public vacancies are considered as AR(1) process such as:

$$\frac{W_t^g}{W_s^g} = \left(\frac{W_{t-1}^g}{W_s^g}\right)^{\rho^g} \exp(\xi_t^{Wg}) \tag{2.64}$$

and

$$\frac{V_t^g}{V_s^g} = \left(\frac{V_{t-1}^g}{V_s^g}\right)^{\rho^g} \exp(\xi_t^{Vg}) \tag{2.65}$$

where ρ^g denotes the duration of the shocks. The terms ξ_t^{Wg} and ξ_t^{Vg} are the white noises associated with the shocks. One can notice that we assume a purely exogenous dynamic of the public wage. In Afonso and Gomes (2014) for instance, the dynamic of the public wage is partly endogenous and function of the dynamic of the real wage. In order to analyze the effects of a rise in public wage everything else equal, we assume a purely exogenous level of public wage.

2.6 Aggregation and market clearing

The market clearing condition can be expressed as:

$$Y_t = C_t + I_t + C^g \tag{2.66}$$

Finally, the following set of equations aggregate the labor market variables:

$$\begin{aligned}
E_t^{tot} &= E_t^p + E_t^g, && (2.67)\\
E_t^g &= (1-\mu)E_t^{og} + \mu E_t^{rg}, && (2.68)\\
E_t^p &= (1-\mu)E_t^{op} + \mu E_t^{rp}, && (2.69)\\
S_t &= S_t^o + S_t^r && (2.70)\\
\theta_t &= \theta_t^p + \theta_t^g && (2.71)
\end{aligned}$$

3 Calibration and strategy

Tables (**??**) and (1) present the baseline calibration of the model and (2) displays the targeted values.

The time discount factor β is set to 0.997, which corresponds to an average annual interest rate of 3%. According to Chetty et al. (2013) and Peterman (2012), we set $-\zeta$ to 1/3 in order to match the macro estimates of the Frisch elasticity of labor supply. Following Smets and Wouters (2003) among others, we set the value of the risk aversion coefficient to $\sigma_c = 2$. $h = 0.33$. The value of the fixed cost of labor market participation is set to $s = 7.5\%$ of the time endowment. This value is halfway between Burnside and Eichenbaum (1996)'s value and Ravn (2005)'s value which are equal to 5% and 9.9% of the time endowment, respectively. The degree of habit formation in consumption is set to $H = 0.85$. Finally, we consider a share of non-Ricardian households $\mu = 0.3$, following Coenen and Straub (2005) for instance.

The Taylor rule's parameters are set at the following usual values: $\alpha^y = 0.5$, $\alpha^\pi = 1.5$ and $\alpha^r = 0.8$.

The share of the public sector *pubshare* is equal to $0,19$. The parameters values for the tax rule are set following Forni et al. (2009) who estimate a New Keynesian model for the Euro Area with a rich fiscal block. Accordingly, we set $\rho^{\tau^c} = 0.96$ and $\rho^{\Omega} = 0.04$. Following Stähler and Thomas (2012) and Afonso and Gomes (2014), the elasticity of matches to unemployment in the public sector is set to $\varphi^g = 0.3$ while the elasticity of matches to unemployment in the private sector is equal to $\varphi^p = 0.5$. Finally, in order to satisfy the Hosios (1990) condition, we set a bargaining power equal to the elasticity of matches to unemployment in the private sector.

Regarding the production side, we set the elasticity of substitution between differentiated goods at $\varepsilon = 7$, which yields an optimal markup of around 17%. The depreciation rate of capital is set to $\delta^k = 0.025$. Finally, the share of capital in the production function is set to a standard value of $\alpha = 0.3$.

Following Michaillat (2014), we assume that the two different states of the business cycle are represented by two different values of unemployment at the steady-state. While Michaillat (2014) choosed to represent this difference of state by imposing different values of real wage, we choose to use directly different values of unemployment. Indeed, economic downturns are represented by a high level of unemployment while economic upturns are represented by a low level of unemployment. This low unemployment rate state consists in $U_s = 6\%$ while the labor market in bad times is represented by $U_s = 12\%$.

The whole economy, and in particular the labor market, changes across the different values of unemployment at the steady state. More precisely, we have $\frac{\partial E_s^{tot}}{\partial U_s} > 0$, $\frac{\partial \theta_s}{\partial U_s} < 0$, $\frac{\partial E_s^i}{\partial U_s} > 0$, $\frac{\partial p_s^i}{\partial U_s} < 0$ and $\frac{\partial q_s^i}{\partial U_s} > 0$.[5] Therefore, at the steady state, a rise in unemployment yields a rise in total employment, a decrease in labor market tightness explained by a decrease in the probability for a worker to find a job and an increase in the probability for a firm to fill its job.

[5] Computations are presented in Appendix A.

Table 1: Baseline calibration

Preferences		
β	0.997	Time-discount factor
$-\zeta$	1/3	Reverse of Frisch elasticity
σ^c	2	Risk aversion
h	0.33	Worked hours
s	0.075h	Fixed cost of participating in the labor market
H	0.85	Degree of Consumption habits
μ	0.3	Share of non-Ricardian workers in the economy
Production		
ε	7	Elasticity of substitution of goods
δ^k	0.025	Depreciation rate of capital
α	0.3	Private sector capital influence
κ^v	0.2	Vacancies posting costs
Monetary Policy		
α^r	0.8	Interest rate smoothing
α^y	0.5	Response coefficient to the output gap
α^π	1.5	Response coefficient to inflation
Fiscal Policy		
ρ^g	0.6	Duration of the fiscal policy shock
ρ^{τ^c}	0.96	VAT AR coefficient
ρ^Ω	0.04	Response coefficient to the debt in GDP share
Labor market and wage bargaining		
κ	2.48	Adjustment cost parameter
η	0.5	Workers' bargaining power
ρ	0.06	Job destruction
φ^p	0.5	Elasticity of matches to unemployment in the private sector
φ^g	0.3	Elasticity of matches to unemployment in the public sector
$pubshare$	0.19	Share of the public sector in the whole economy

4 Results

4.1 The effects of fiscal policy on the labor market and output over the business cycle

For all simulations in this paper we use the Dynare program created by the CEPREMAP team. The algorithm used by Dynare for the second order

Table 2: Targeted Values

π_s	1	Inflation
p_s	1	Prices
Y_s	1	Output
C^g	0.2	Public Consumption
b	$0.3Y_s$	Unemployement benefit
τ^c	0.20	VAT
τ_s^w	0.16	Income tax
U_s	0.08	Unemployment
q_s^p	0.7	Job filling probability in private sector
q_s^g	0.8	Job filling probability in public sector
Ψ_s	$0.6Y_s$	Debt target

approximation of our model is very close to the one developed in Schmitt-Grohé and Uribe (2004). In addition, simulations are carried out by using the pruning method, in order to avoid triggering polynomials of increasing degrees when simulating the model.[6].

For the two shocks considered, we find a similar result: fiscal policies have a greater effect on employment, unemployment and output in the case of the high steady-state value for the unemployment rate. As we will see throughout this section, these results are driven by two main elements: a wider pool of job seekers and the crucial role of the wage dynamic.

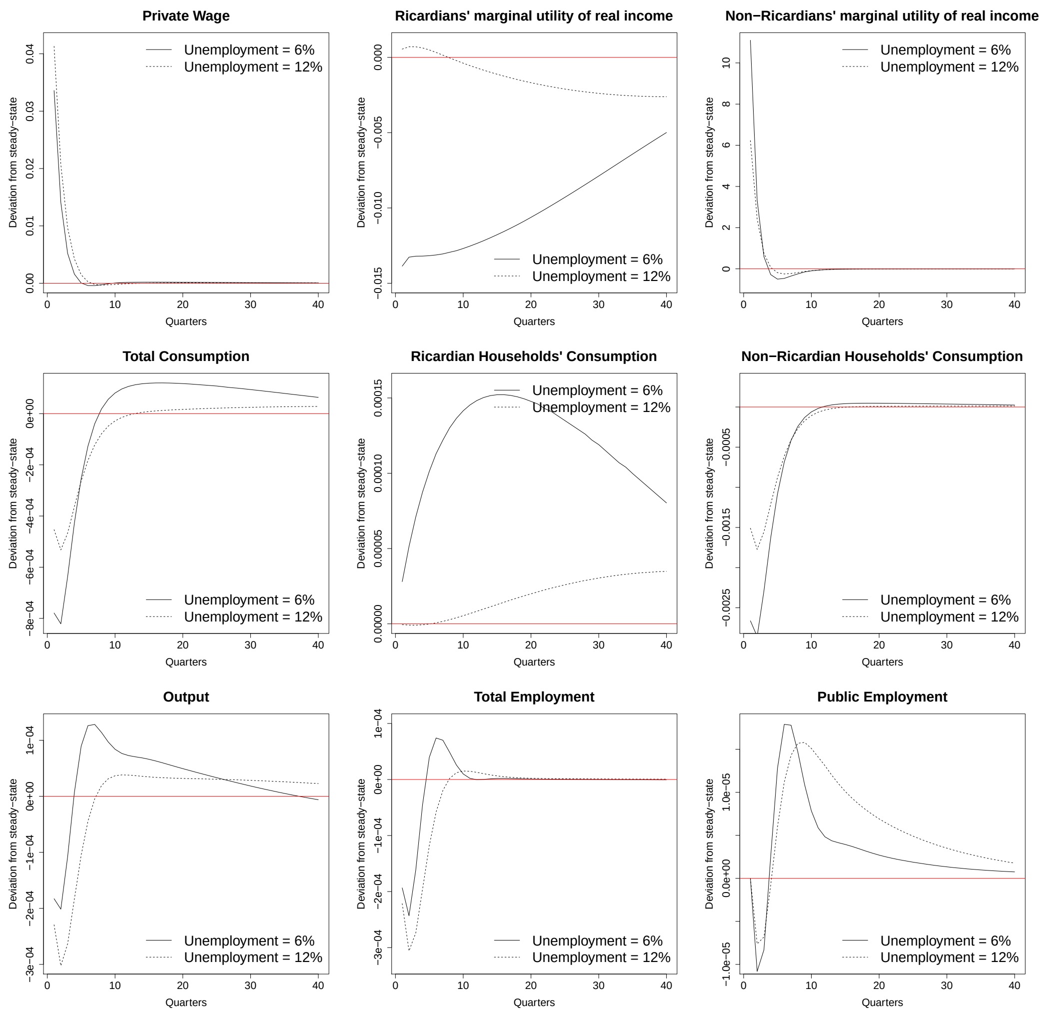
4.1.1 The effets of a cut in public-sector wage:

Impulse response functions for a cut of 1% in public-sector wage are displayed in Figures 1 and 2. A first observation is that the response of the economy greatly differs according to the steady-state unemployment rate. Before explaining these non-linear effects, let us focus on the general effects of a cut in the public-sector wage.

A drop in the public-sector wage triggers an automatic decrease in consumption of the non-Ricardian households. This effect is amplified by a decrease in total employment and is not compensated by the increase in the private sector wage. This negative effect on demand produced a decline in output in short run. On the contrary, consumption of Ricardian households

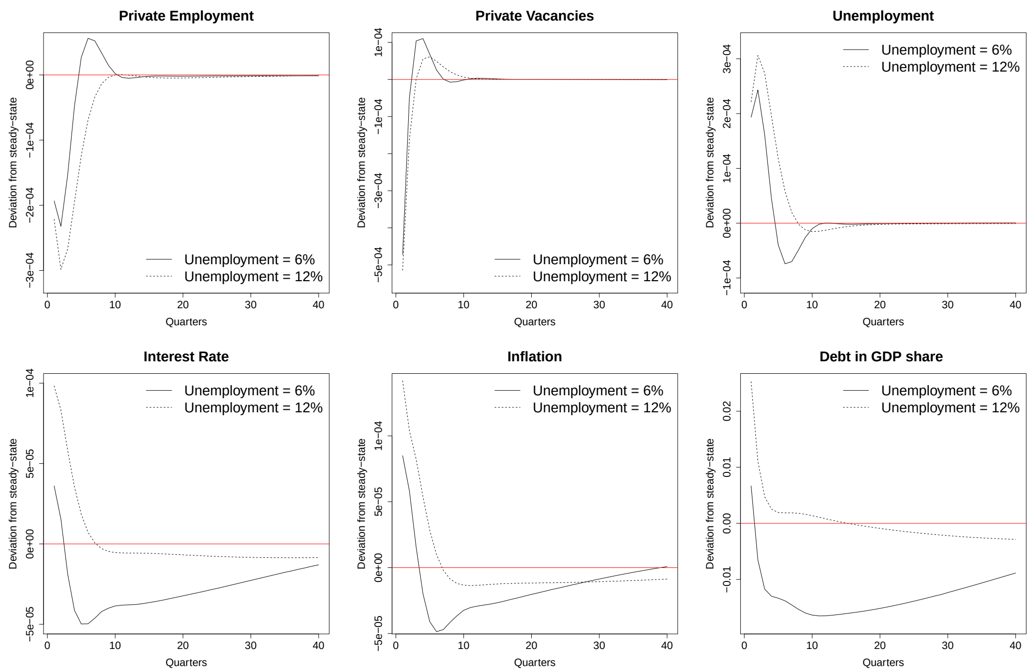
[6]See for instance Lombardo and Uhlig (2014) for a presentation of the pruning method.

Figure 1: Response to a cut of 1% in public sector wage I



tends to increase, regardless of steady-state unemployment. This increase in Ricardian's consumption is driven by two main transmission channels. First, the cut in public-sector wage puts a downward pressure on the real interest rate (especially in the case of $U_s = 12\%$), which triggers a positive wealth effect for this class of households. Second, and this is only valid for $U_s = 6\%$, VAT decreases following the decline in debt, so that it puts additional upward pressures on consumption of Ricardian households. Despite the rise in consumption of Ricardian households, total consumption and output de-

Figure 2: Response to a cut of 1% in public sector wage II



crease in the short run.

Following this negative short-run effect on private activity, private employment tends to decrease. This fall in private employment positively affects the marginal productivity of labor and puts upward pressure on the private-sector wage for few periods. However, two additional effects tends to ease this dynamics on private wage. First, a decrease in public wage has a direct negative effect on the private-sector wage, as shown in equation (2.59). From the workers' point of view, the public sector becomes less attractive so that a part of the pool of the job seekers turns toward the private sector and will accept to work for a lower real wage. Second, a decrease in the public-sector wage diminishes the value to be unemployed, which puts additional downward pressures on the private-sector wage. As a consequence, the private real wage increases but only for few periods.

As said previously, the effects of the cut in the public-sector wage are greatly influenced by the level of unemployment at the time of the shock.

Before going into a detailed explanation, this restrictive fiscal policy has larger and sizable negative effects on employment and output during economic downturn than during economic upturn. As a consequence, while a reduction in public-sector wage decreases debt in GDP share in the case of $U_s = 6\%$, debt increases for about fifteen periods before being reduced in the more long run in the case of $U_s = 12\%$.

Impulse response functions show that the effects on private employment of the decline in public-sector wage are greater when the steady-state unemployment rate is large and that the real wage rises more sharply in this case. Closely to Michaillat (2014), the larger the pool of job seekers at the steady-state, the larger the effects of a negative demand shock on employment. As a consequence and all things being equal, the negative fiscal policy shock induces a stronger fall in private employment when unemployment is high at the time of the shock. This larger decrease in private employment induces a greater rise in the marginal productivity of labor: as a result, upward pressures on the private-sector wage are greater with $U_s = 12\%$.

This larger rise in the private real wage has contradictory effects on private consumption. First, despite a stronger degradation of private employment, consumption of non-Ricardian households is less reduced in this case. However, larger upward pressures on the real wage trigger a higher response of inflation in the case of $U_s = 12\%$. As a consequence, the real interest rate is significantly less reduced and its response is even slightly positive in the long-run. Therefore, the positive crowding-out effect of a fall in public wage on consumption of Ricardian households is dimished is a sizeable way. This effect on Ricardians' consumption prevails over the lower degradation of consumption of the non-Ricardian households so that the degradation of total private consumption is significantly lower when public wage is reduced in times of low unemployment. As a result, this restrictive fiscal policy has larger negative effects on output when it is implemented in periods of high unemployment.

A fall in public-sector wage allows to reduce importantly the debt in GDP share when $U_s = 6\%$. However, debt increases in the short-run before being reduced only slightly whit $U_s = 12\%$.

4.1.2 The effects of a cut in public vacancies

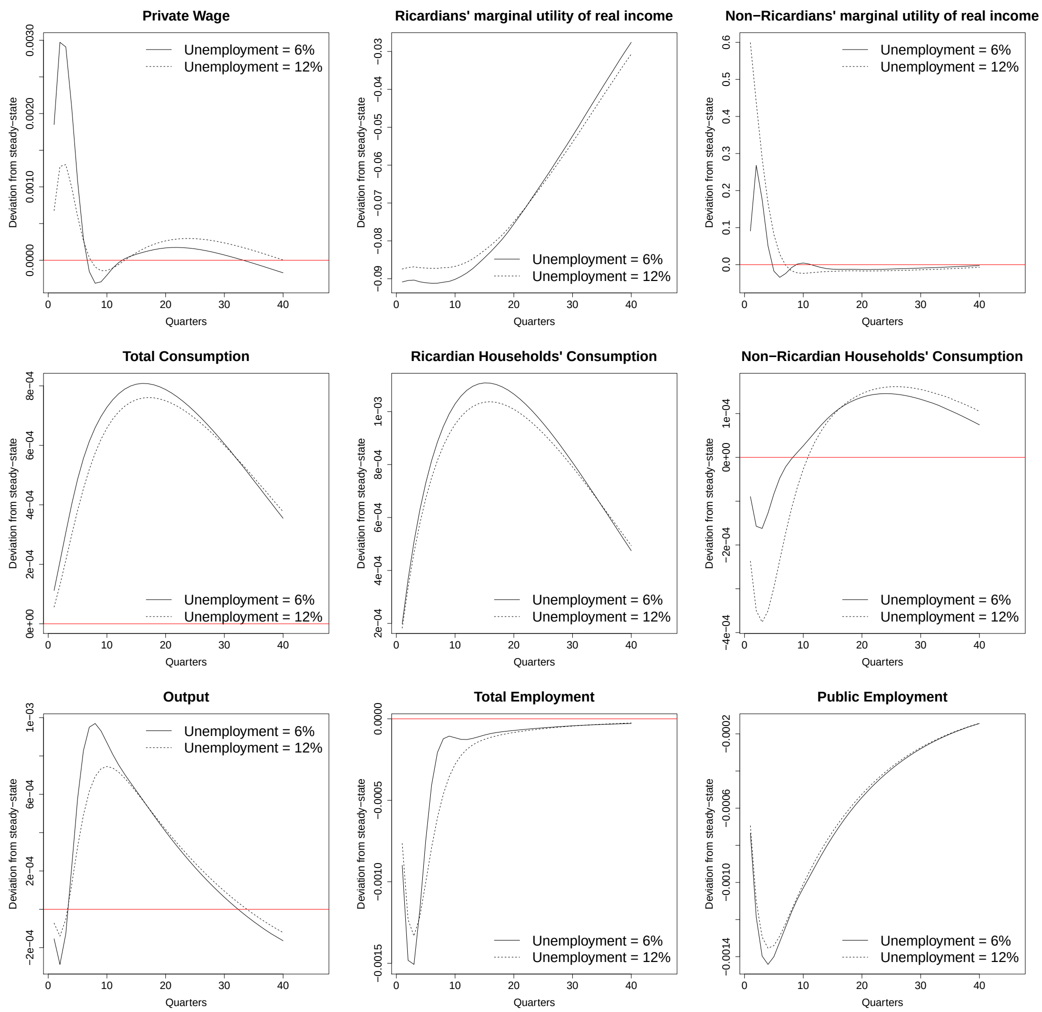
Before investigating the non-linear effects of this cut in public expenditure, let us describe the overall effects of a cut in public vacancies in this model. A first observation is that decreasing public employment triggers a positive response of output. As said previously, the implied negative output fiscal multiplier is due to the assumption of an unproductive public sector. To add public sector in total GDP would produce a decline in output. However, the present article does not focus on the size of the output fiscal multiplier but rather focuses on the impact of the initial unemployment rate on the size of the output fiscal multiplier. Response of output in this model has to be considered as the response of private activity to changes in the fiscal stance. Impulse response functions for a cut of 1% in public-sector vacancies are displayed in Figures 3 and 4.

A decrease in public-sector employment triggers an automatic positive crowding-out effect on the private-sector labor market. Following the contraction of public employment, the private sector takes advantage of a larger pool of job seekers so that the number of matches in this sector tends to increase. However, the response of total employment remains negative.

Consumption of non-Ricardian households decreases following the drop in public employment. However, impulse response functions indicate that consumption of non-Ricardian households goes up in the medium run. Three elements drive this positive response. First, the rise in private employment offsets partly the fall in public employment. Second, the positive response of private activity and employment puts an upward pressure on private-sector real wage. Third, VAT falls following the decline in debt in GDP share. Overall, consumption of non-Ricardian households increases in the mid and long run.

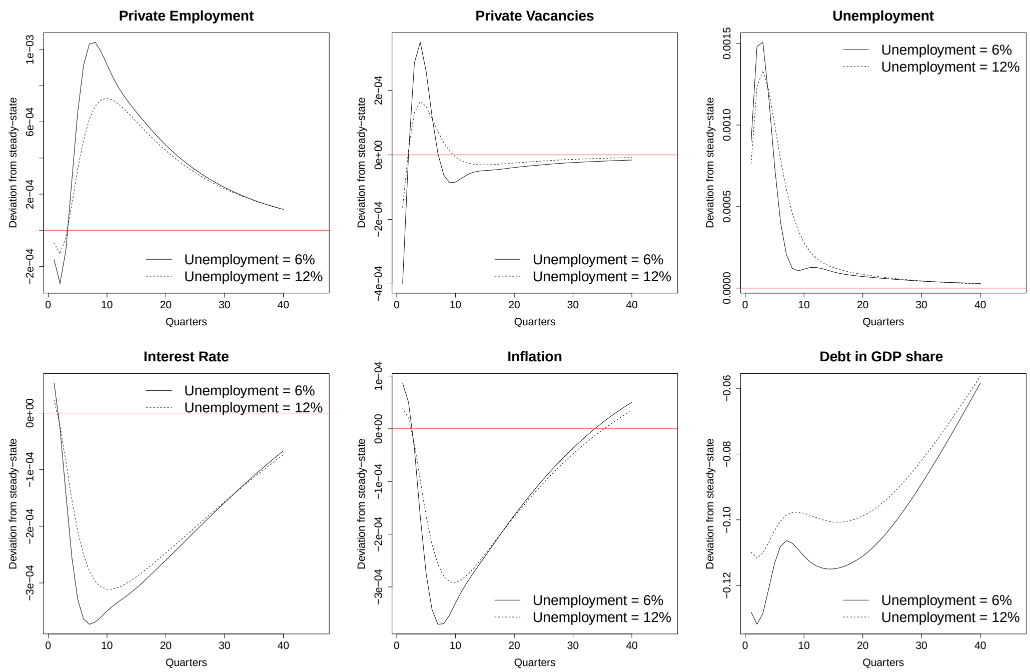
Consumption of Ricardian households also reacts positively to the drop in public vacancies. Similarly to non-Ricardian households, the fall in VAT puts an upward pressure on consumption of Ricardian households. Moreover, the overall drop in prices and the implied decrease in the real interest rate boost Ricardian consumption. As said previously, a tightening of the public sector crowds in private activity.

Figure 3: Response to a cut of 1% in public vacancies I



Impulse response functions indicate that, similarly to a cut in the public-sector wage, the response of output and employment is greater when the steady-state unemployment is high. The transmission channel at work is very close to the previous case. With a larger pool of job seekers, the positive crowding-out effect of a cut in public vacancies on the private-sector labor market is amplified. This mechanism is perfectly similar to what is demonstrated in Michaillat (2014).

Figure 4: Response to a cut of 1% in public vacancies II



Consumption of non-Ricardian households is better (less negative) when the unemployment rate is low ($U_s = 6\%$) at the time of the shock. It can be explained by three reasons: when $U_s = 6\%$, the cut in public vacancies triggers a better response of total employment, a larger rise in the private-sector wage and a greater decline in VAT due to a lower fall in debt.

As said above, when the steady-state unemployment is low, the private-sector real wage increases more sharply. Similarly to the cut in public wage, the greater response of private employment when $U_s = 6\%$ generates a lower marginal productivity of labor than in the case of a high steady-state unemployment. As a consequence, prices and then the real interest rate are lower in this scenario. Hence, the response of consumption of non-Ricardian households is greater in the case of $U_s = 6\%$.

The combination of a better response of non-Ricardian households and of Ricardian househoulds triggers a larger rise in output when the unemployment rate is low at the time of shock. As said previously, the cut in public employment enables a more sizeable decline in debt in GDP share in this case.

4.1.3 Overall remarks

To summarize the results, we attempt to show in this paper that cuts in public-sector employment and salaries are more harmful in terms of output and total employment when unemployment is already large. The larger cost on total employment is similar to what highlights Michaillat (2014) since the effect of the public sector to the private one is amplified by the presence of a larger pool of job seekers at the steady-state.

Unlike Sims abd Wolff (2013), it is also important to notice that the more positive response of consumption of the Ricardians is not due in our model to a higher marginal utility of consumption in economic downturns. The authors highlight this transmission channel for explaining different output fiscal multipliers over the business cycle. This is not the case in our model according to the definition of the steady-states. The value of Ricardian consumption at the steady state is obtained residually with the steady-state value of non-Ricardian consumption such as:

$$C_s^o = \frac{C_s - \mu C_s^r}{1-\mu}, \tag{4.1}$$

whith C_s^o, C_s^r and C_s respectively the steady-state value of C_t^o, C_t^r and C_t.

The steady-state value of non-Ricardian consumption is lower with $U_s = 12\%$ since real wage is larger than unemployment benefits at the steady state. It triggers a higher marginal utility of consumption for this class of households but it has no impact on their consumption behavior since they simply consume their disposable income. However, a lower level of consumption at the steady state for the non-Ricardian households implies a higher consumption for the Ricardians in bad times so that the the transmission channel highlighted in Sims and Wolff (2013) is not present in our model.

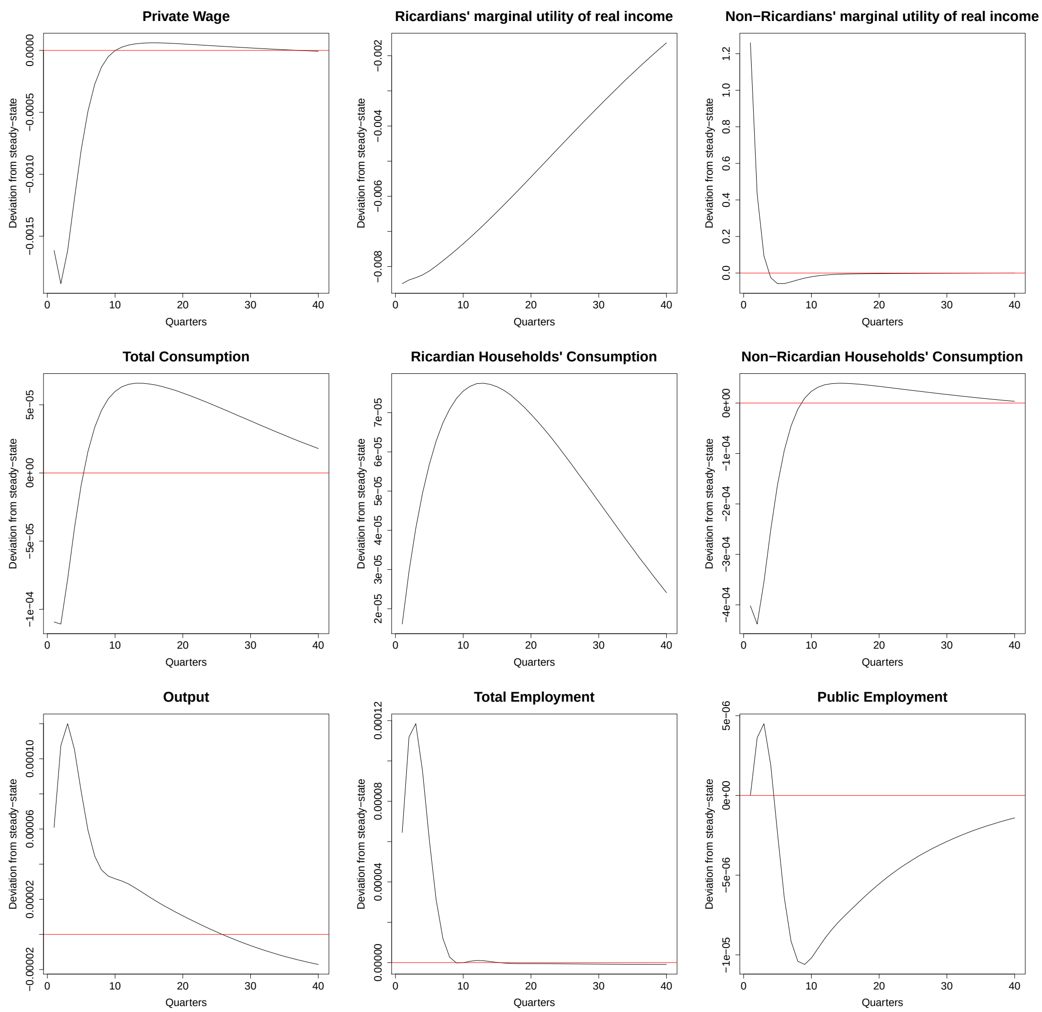
4.2 On the importance of the composition of the fiscal adjustment

Only few articles have recently investigated the effects of changes in public wage and one notable exception is Afonso and Gomes (2014). Results in our article regarding the response of the economy to a public wage shock are partly in opposition with those of Afonso and Gomes (2014). Our model predicts a rise in the private-sector wage and a drop in employment following a cut in the public wage while Afonso and Gomes (2014) argue for an opposite dynamic. The authors demonstrate that the private wage increases and that employment falls following a rise in public-sector wage. Impulse response functions are displayed in Figures 5 and 6.

First, a higher public wage increases the value of being unemployed, which is also included in our definition of the private-sector real wage. Secondly, their model generates a rise in marginal productivity of labor which creates upward pressures on private real wage. On the contrary, in our model a cut in the public wage trigger a unambiguous rise in the marginal productivity of labor thanks to a positive effect on output and a fall in employment. This difference partly explains the different dynamic of the private-sector real wage produced by our model following a public wage shock. It is important to notice that this difference in the response of the marginal productivity of labor can be explained by the fact that the public sector is productive in Afonso and Gomes (2014) since labor in the public sector serves to produce public goods. As said previously, the public sector is not taken into account in our definition of GDP so that the positive response of output lies in rise in private activity.

Moreover, we differ from Afonso and Gomes (2014) since in our model the government is allowed to issue nominal debt each period and VAT is assumed to be adjusted to ensure public finance sustainability in the long run. In Afonso and Gomes (2014), the authors assume that the wage bill is entirely funded by the labor income tax, so that the budget is balanced each period. For comparison purposes, we modify the fiscal block such as the labor income tax is adjusted to maintain a balanced budget. The new budget constraint is then:

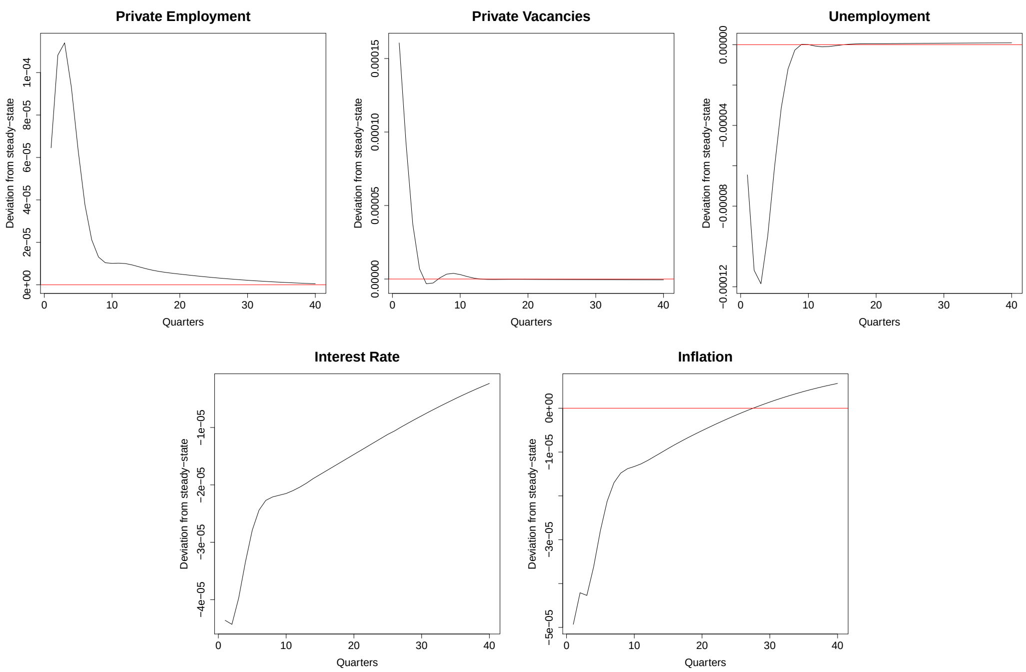
Figure 5: Response to a cut in public sector wage according to the composition of the fiscal adjustment I



$$\tau^c(C_t^o + C_t^r) + \tau_t^w W_t^p E_t^p h + E_t^p W_t^p h = C^g + (1-\tau_t^w)(W_t^g E_t^g h) + bS_t. \tag{4.2}$$

where τ^c is assumed to be constant. In this scenario, the labor income tax reacts contemporaneously to a cut in the public wage to ensure a balanced budget. As shown in Appendix **??**, our model reproduces similar results

Figure 6: Response to a cut in public sector wage according to the composition of the fiscal adjustment II



in this case. In the case of a negative shock on public wage, employment rises, unemployment falls and private real wage decreases. We emphasize that the composition of the fiscal adjustment can alter greatly the results. According to Afonso and Gomes (2014), the effects of a change in the labor income tax on the private-sector real wage is ambiguous. The authors argue that a rise in the labor income tax lowers the match surplus and then puts an upward pressure on the private real wage. On the contrary, the match surplus going to the worker is reduced, which tends to rise the private-sector wage. According to our simulations, a drop in the labor income tax reduces the private-sector wage through a large positive effect on the match surplus.

5 Conclusion

This paper attempts to investigate the non-linear effects of fiscal policy over the business cycle with a focus on public-sector employment and salaries. The main result is that cuts in public employment and wages are more harmful in terms of output and employment in periods of high unemployment. Large cuts in government expenditure have been implemented in the Euro Area from 2010, a period of historically high unemployment. This paper argues, alongside numerous articles, for large contractionary effects of the austerity plans in the aftermath of the crisis. First, we show that cuts in the public-sector labor market have stronger negative effects on employment in periods of high unemployment rate. Second, contractionary effects on output are also magnified when unemployment is high at the time of the implementation of a restrictive fiscal policy. Likewise, the effectiveness of these austerity plans to reduce deficit and debt was weak because of a large cost on economic activity.

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A How are labor market variables impacted by unemployment at steady state ?

Given that $E_s^{tot} = 1 - U_s$ and $S_s = U_s + \rho E_s^{tot}$, we have

$$E_s^{tot} = 1 - (\rho E_s^{tot} - S_s)$$
$$\Leftrightarrow E_s^{tot} = \frac{1 + S_s}{1 + \rho} \tag{A.1}$$

One can easily note that:

$$\frac{\partial E_s^{tot}}{\partial S_s} = \frac{1}{1+\rho} > 0 \tag{A.2}$$

Moreover, by definition:

$$\frac{\partial \theta_s}{S_s} = -\frac{V_s^p}{(S_s)^2} - \frac{V_s^g}{(S_s)^2} < 0 \tag{A.3}$$
$$\tag{A.4}$$

From equation (2.3), we have:

$$E_s^p = (1-\rho)E_s^p + p_s^p(1-\rho)S_s$$
$$\Leftrightarrow E_s^p = \frac{(1-\rho)p_s^p S_s}{\rho} \tag{A.5}$$

From equations (2.5) and (2.6), we have:

$$p_s^p = \kappa_e^p (S_s)^{(\varphi^p - 1)} (V_s^p)^{(1-\varphi^p)} \tag{A.6}$$

Given equation (A.6), we can define E_s^p as:

$$E_s^p = \left(\frac{1-\rho}{\rho}\right) \kappa_e^p (S_s)^{\varphi^p} (V_s^p)^{(1-\varphi^p)} \tag{A.7}$$

Thus, we have:

$$\frac{\partial E_s^p}{\partial S_s} = \left(\frac{1-\rho}{\rho}\right) \kappa_e^p \varphi^p (S_s)^{\varphi^p - 1} (V_s^p)^{(1-\varphi^p)} > 0 \tag{A.8}$$

Similarly, one can define E_s^g and p_s^g as:

$$p_s^g = \kappa_e^g (S_s)^{(\varphi^g - 1)} (V_s^g)^{(1-\varphi^g)} \tag{A.9}$$

and

$$E_s^g = \left(\frac{1-\rho}{\rho}\right) \kappa_e^g (S_s)^{\varphi^g} (V_s^g)^{(1-\varphi^g)}. \tag{A.10}$$

Thus, we have

$$\frac{\partial E_s^g}{\partial S_s} = \left(\frac{1-\rho}{\rho}\right) \kappa_e^g \varphi^p (S_s)^{\varphi^g - 1} (V_s^g)^{(1-\varphi^g)} > 0 \tag{A.11}$$

Thanks to equation A.6, one can note that

$$\frac{\partial p_s^p}{\partial S_s} = \kappa_e^p (\varphi^p - 1)(S_s)^{\varphi - 2} (V_s^p)^{1-\varphi^p} < 0 \tag{A.12}$$

Thanks to equation 2.7, we can define the probability for a firm to find a worker at the steady state such as

$$q_s^p = \kappa_e^p \left(\frac{V_s^p}{S_s}\right)^{-\varphi^p} \tag{A.13}$$

Thus, we have

$$\frac{\partial q_s^p}{\partial S_s} = \kappa_e^p \varphi^p S_s^{\varphi - 1} V_s^{p-\varphi} > 0 \tag{A.14}$$

Finally, accordingly to equation (2.62), nominal debt at the steady-state is equal to:

$$B_s = \Psi_s Y_s P_s. \tag{A.15}$$

B Wage equation calculation

We start from the surplus' optimal sharing rule given by the equation (2.58). Knowing that:

$$\frac{\partial \Upsilon_t}{\partial W_t^p} = (1-\mu)(1-\tau_t^w)\lambda_t^{rio}h + \mu(1-\tau_t^w)\lambda_t^{rir}h, \tag{B.1}$$

and

$$\frac{\partial \Upsilon_t^{E_f}}{\partial W_t^p} = -h, \tag{B.2}$$

and after giving to Υ_t and $\lambda_t^{E_f}$ their respective value described by equations (2.54) and (2.45), (2.58) yields:

$$\begin{aligned}
&\eta\left[(1-\mu)(1-\tau_t^w)\lambda_t^{rio} + \mu(1-\tau_t^w)\lambda_t^{rir}\right] \\
&\times\left[(1-\alpha)\frac{Y_t}{E_t^p} - W_t^p h + (1-\rho)\beta_{t,t+1}\lambda_{t+1}^{E_f}\right] \\
= (1-\eta)&\Bigg\{\mu\Bigg[(1-\tau_t^w)\lambda_t^{rir}W_t^p h - \lambda_t^{rir}b + \frac{(1-h-s)^{1-\zeta}-(1-s)^{1-\zeta}}{1-\zeta} \\
&+\beta E_t\left[(1-\rho)(1-p_t^p)(\lambda_{t+1}^{E_{rp}}-\lambda_{t+1}^{S_r}) - p_t^g(1-\rho)(\lambda_{t+1}^{E_{rg}}-\lambda_{t+1}^{S_r})\right]\Bigg] \\
&+(1-\mu)\Bigg[(1-\tau_t^w)\lambda_t^{rio}W_t^p h - \lambda_t^{rio}b + \frac{(1-h-s)^{1-\zeta}-(1-s)^{1-\zeta}}{1-\zeta} \\
&+\beta E_t\left[(1-\rho)(1-p_t^p)(\lambda_{t+1}^{E_{op}}-\lambda_{t+1}^{S_o}) - p_t^g(1-\rho)(\lambda_{t+1}^{E_{og}}-\lambda_{t+1}^{S_o})\right]\Bigg]\Bigg\}
\end{aligned}$$

$$\begin{aligned}
&\Leftrightarrow (1-\tau_w)(\mu\lambda_t^{rir} + (1-\mu)\lambda_t^{rio})W_t^p h \\
&= \eta(1-\tau_t^w)(\mu\lambda_t^{rir} + (1-\mu)\lambda_t^{rio})\left[\frac{(1-\alpha)Y_t}{E_t^p} + (1-\rho)\beta_{t,t+1}\lambda_{t+1}^{E_f}\right] \\
&+(1-\eta)\left[[\mu\lambda_t^{rir} + (1-\mu)\lambda_t^{rio}]b + \frac{(1-s)^{1-\zeta}-(1-h-s)^{1-\zeta}}{1-\zeta}\right] \\
&-(1-\eta)(1-\rho)(1-p_t^p)\beta E_t[\Upsilon_{t+1}] \\
&+(1-\eta)(1-\rho)p_t^g\beta E_t[\mu(\lambda_{t+1}^{E_{rg}}-\lambda_{t+1}^{S_r}) + (1-\mu)(\lambda_{t+1}^{E_{og}}-\lambda_{t+1}^{S_o})]
\end{aligned}$$

Moreover, since equation (2.58) yields:

$$\beta E_t[\Upsilon_{t+1}] = \frac{\eta}{(1-\eta)}E_t\left[\beta_{t,t+1}(1-\tau_{t+1}^w)(\mu\lambda_{t+1}^{rir} + (1-\mu)\lambda_{t+1}^{rio})\lambda_{t+1}^{E_f}\right],$$

we finally obtain:

$$
\begin{aligned}
&\Leftrightarrow (1-\tau_w)(\mu\lambda_t^{rir}+(1-\mu)\lambda_t^{rio})W_t^p h \\
&= \eta(1-\tau_t^w)(\mu\lambda_t^{rir}+(1-\mu)\lambda_t^{rio})\left[\frac{(1-\alpha)Y_t}{E_t^p}+(1-\rho)E_t[\beta_{t,t+1}\lambda_{t+1}^{E_f}]\right] \\
&\quad +(1-\eta)\left[(\mu\lambda_t^{rir}+(1-\mu)\lambda_t^{rio})b+\frac{(1-s)^{1-\zeta}-(1-h-s)^{1-\zeta}}{1-\zeta}\right] \\
&\quad -\eta(1-p_t^p)(1-\rho)E_t\left[\beta_{t,t+1}(1-\tau_{t+1}^w)(\mu\lambda_{t+1}^{rir}+(1-\mu)\lambda_t^{rio})\lambda_{t+1}^{E_f}\right] \\
&\quad +(1-\eta)(1-\rho)p_t^g\beta E_t[\mu(\lambda_{t+1}^{E_{rg}}-\lambda_{t+1}^{S_r})+(1-\mu)(\lambda_{t+1}^{E_{og}}-\lambda_{t+1}^{S_o})]
\end{aligned}
$$

$$
\begin{aligned}
(1-\tau_t^w)W_t^p h &= \eta(1-\alpha)(1-\tau_t^w)\frac{Y_t}{E_t^p}+(1-\eta)\left[b+\frac{(1-s)^{1-\zeta}-(1-h-s)^{1-\zeta}}{(1-\zeta)\mu\lambda_t^{rir}+(1-\mu)\lambda_t^{rio}}\right] \\
&\quad +\eta(1-\rho)E_t\left\{\beta_{t,t+1}\left[1-(1-p_t^p)(1-\tau_{t+1}^w)\tilde{\Lambda}_{t+1}\right]\lambda_{t+1}^{E_f}\right\} \\
&\quad +(1-\eta)(1-\rho)p_t^g\beta E_t[\Lambda_t(\lambda_{t+1}^{E_{rg}}-\lambda_{t+1}^{S_r})+(1-\Lambda_t)(\lambda_{t+1}^{E_{og}}-\lambda_{t+1}^{S_o})]
\end{aligned}
\tag{B.3}
$$

C Steady-State calculations

Starting from the long-run targeted values described in table 2, we now describe the steady-state calculations. We first assume that $W_s^g = W_s^p$.

From equation (2.2), one can easily define the value of total employment at the steady-state such as:

$$E_s^{tot} = 1 - U_s. \tag{C.1}$$

From equation (2.2), the number of job seekers in the economy as a whole is equal to:

$$S_s = U_s + \rho E_s^{tot}. \tag{C.2}$$

By definition, assuming that *pubshare* is the size of the public sector on the labor market, we can define the value of public employment as

$$E_s^g = E_s^{tot} \times pubshare. \tag{C.3}$$

Then, from equations (C.3) and (2.67), we define the value of private employment at the steady state as:

$$E_s^p = E_s^{tot} - E_s^g. \tag{C.4}$$

By definition we have:

$$E_s^r = \mu E_s^{tot} \tag{C.5}$$
$$\text{and} \quad E_s^o = (1-\mu) E_s^{tot} \tag{C.6}$$

Thanks to equation (2.40), we can define

$$V_s^p = \rho \frac{E_s^p}{q_s^p} \tag{C.7}$$

and we assume similarly that

$$V_s^g = \rho \frac{E_s^g}{q_s^g}. \tag{C.8}$$

Joining the matching functions and the definition of the probability for a firm to fill its job, described by the equations (2.5) and (2.7) we are able to define the matching technology in each sector as:

$$\kappa_e^p = \frac{V_s^p q_s^p}{S_s^{\varphi^p} (V_s^p)^{1-\varphi^p}} \tag{C.9}$$
$$\kappa_e^g = \frac{V_s^g q_s^g}{S_s^{\varphi^g} (V_s^g)^{1-\varphi^g}} \tag{C.10}$$

Thanks to the previous equations and to the equations (2.5), we can define the number of matches in each sector at the steady state as

$$M_s^p = \kappa_e^p S_s^{\varphi^p} (V_s^p)^{1-\varphi^p} \tag{C.11}$$
$$\text{and} \quad M_s^g = \kappa_e^g S_s^{\varphi^g} (V_s^g)^{1-\varphi^g}. \tag{C.12}$$

Thanks to equations (C.2), (C.11) and (C.12), we can define the probability for a worker to find a job in each sector at the steady state as

$$p_s^p = \frac{M_s^p}{S_s} \tag{C.13}$$
$$\text{and} \quad p_s^g = \frac{M_s^g}{S_s}. \tag{C.14}$$

According to equation (2.24) we have:

$$R_s^k = r_s + \delta^k - 1. \tag{C.15}$$

We assume that at the steady-state, marginal cost is equal to the desired (flexible prices) markup such as:

$$mc_s = \frac{\varepsilon}{\varepsilon - 1}. \tag{C.16}$$

Thanks to the previous equations and using equation (2.47), we can define the marginal cost of labor at the steady state such as:

$$x_s = (1-\alpha) mc_s \left(\frac{Y_s}{E_s^p h} \right) - W_s^p h). \tag{C.17}$$

From equation (2.25) and the definition of $S\left(\frac{I_t^o}{I_{t-1}^o}\right)$, the steady-state of Tobin's Q is:

$$Q_s = 1. \tag{C.18}$$

According to equation (2.46), we have:

$$k_s = \alpha mc_s \frac{Y_s}{R_s^k}, \tag{C.19}$$

while from aggregation we have:

$$k_s^o = \frac{k_s}{(1-\mu)} \tag{C.20}$$

$$\text{and} \quad I_s^o = \frac{I_s}{(1-\mu)}. \tag{C.21}$$

Thanks to the equation (2.41), we can define the TPF at the steady-state as:

$$\epsilon_s^a = \frac{Y_s}{k_s^\alpha (E_s^p h)^{1-\alpha}}. \tag{C.22}$$

According to the market clearing condition defines by equation (2.66), we have

$$C_s = Y_s - C^g - Is. \tag{C.23}$$

The definition of the LMT given by equation (2.8) yields

$$\theta_s^p = \frac{V_s^p}{S_s} \tag{C.24}$$

$$\text{and} \quad \theta_s^g = \frac{V_s^g}{S_s}. \tag{C.25}$$

Aggregation yields:

$$\theta_s = \theta_s^p + \theta_s^g. \tag{C.26}$$

By construction, we have:

$$q_s^1 = \frac{\lambda_s^{rio} Y_s mc_s}{1 - \beta\theta^p \pi_s^{\epsilon-1}} \tag{C.27}$$

$$q_s^2 = \frac{\lambda_s^{rio} Y_s}{1 - \beta\theta^p \pi_s^{\epsilon-1}}, \tag{C.28}$$

and thanks to equation (2.52):

$$p_s^{opt} = \frac{\varepsilon}{\varepsilon - 1} \frac{q_s^1}{q_s^2}. \tag{C.29}$$

The value of a job at the steady-state for a firm is equal to:

$$\lambda_s^{Ef} = \frac{1-\alpha}{1-(1-\rho)\beta} \frac{Y_s}{E_s^p} - \frac{1}{1-(1-\rho)\beta} W_s^p h. \tag{C.30}$$

Thanks to the previous equations we can now define the value of posting a vacancy:

$$\kappa^v = \beta \left(\frac{(1-\alpha)Y_s}{E_s^p} - W_s^p h + (1-\rho)\beta\lambda_s^{Ef} \right) q_s^p. \tag{C.31}$$

The utility function of the union at the steady state can be defined as:

$$\Upsilon_s = (1-\mu)(\lambda_s^{E_{op}} - \lambda_s^{S_o}) + \mu(\lambda_s^{E_{rp}} - \lambda_s^{S_r}). \tag{C.32}$$

Finally, by definition,

$$mpl_s = \frac{(1-\alpha)Y_s}{E_s^p h}. \tag{C.33}$$

Marginal utility of real income in terms of non-Ricardian consumption

If we admit that $W_s^g = W_s^p$, the non-Ricardian consumption at the steady state can be expressed as

$$C_s^r = \{(1-\tau_s^w)[E_s^r W_s^p h + (1-E_s^r)b]\}^(1+\tau_s^c) \tag{C.34}$$

We express the Ricardians' consumption at the steady state in terms of wage as

$$C_s^o = \frac{C_s - muC_s^r}{1-mu} \tag{C.35}$$

Then, the marginal utility of real income for Ricardian and non-Ricardian households can be expressed as

$$\lambda_s^{rio} = \frac{1-\beta H}{1+\tau_s^c}[(1-H)C_s^o]^{-\sigma_c}$$
$$\Leftrightarrow \lambda_s^{rio} = \frac{1-\beta h}{1+\tau_s^c}\left\{(1-H)\frac{1}{1-\mu}\left\{C_s - \frac{\mu}{1+\tau_s^c}[(1-\tau_s^w)E_s^r W_s^p h + (1-E_s^r)b]\right\}\right\}^{-\sigma_c} \tag{C.36}$$

$$\lambda_s^{rir} = (1-\beta H)[(1-H)C_s^r]^{-\sigma_c}$$
$$\Leftrightarrow \lambda_s^{rir} = \frac{1-\beta H}{1+\tau_s^c}\{(1-H)\{(1-\tau_s^w)[E_s^r W_s^p h + (1-E_s^r)b]\}\}^{-\sigma_c} \tag{C.37}$$

Workers' marginal utilities in terms of unemployment marginal utility

For Ricardian workers

$$\lambda_s^{E_{op}} = (1-\tau_s^w)\lambda_s^{rio}W_s^p h - \frac{1-(1-h-s)^{1-\zeta}}{1-\zeta} + (1-\rho)\beta\lambda_s^{E_o p} + \rho\beta\lambda_s^{S_o}$$
$$\Leftrightarrow [1-(1-\rho)\beta]\lambda_s^{E_{op}} = (1-\tau_s^w)\lambda_s^{rio}W_s^p h - \frac{1-(1-h-s)^{1-\zeta}}{1-\zeta} + \rho\beta\lambda_s^{S_o}$$
$$\Leftrightarrow \lambda_s^{E_{op}} = \frac{1}{1-(1-\rho)\beta}\left[(1-\tau_s^w)W_s^p h\lambda_s^{rio} - \frac{1-(1-h-s)^{1-\zeta}}{(1-\zeta)} + \beta\rho\lambda_s^{S_{os}}\right] \tag{C.38}$$

$$\lambda_s^{E_{og}} = (1-\tau_s^w)\lambda_s^{rio} W_s^g h - \frac{1-(1-h-s)^{1-\zeta}}{1-\zeta} + (1-\rho)\beta\lambda_s^{E_og} + \rho\beta\lambda_s^{S_o}$$
$$\Leftrightarrow [1-(1-\rho)\beta]\lambda_s^{E_{og}} = (1-\tau_s^w)\lambda_s^{rio} W_s^g h - \frac{1-(1-h-s)^{1-\zeta}}{1-\zeta} + \rho\beta\lambda_s^{S_o}$$
$$\Leftrightarrow \lambda_s^{E_{og}} = \frac{1}{1-(1-\rho)\beta}\left[(1-\tau_s^w)W_s^g h\lambda_s^{rio} - \frac{1-(1-h-s)^{1-\zeta}}{(1-\zeta)} + \beta\rho\lambda_s^{S_{os}}\right]$$
$$\Leftrightarrow \lambda_s^{E_{og}} = \frac{1}{1-(1-\rho)\beta}\left[(1-\tau_s^w)W_s^p h\lambda_s^{rio} - \frac{1-(1-h-s)^{1-\zeta}}{(1-\zeta)} + \beta\rho\lambda_s^{S_{os}}\right] \tag{C.39}$$

$$\lambda_s^{S_o} = b\lambda_s^{rio} - \frac{1-(1-s)^{1-\zeta}}{1-\zeta} + (1-p_s^p-p_s^g)\beta\lambda_s^{S_o} + \rho(p_s^p+p_s^g)\beta\lambda_s^{S_o}$$
$$+(1-\rho)\beta[p_s^p\lambda_s^{E_{op}} + p_s^g\lambda_s^{E_{og}}]$$
$$\Leftrightarrow \lambda_s^{S_o}[1-\beta+\beta(1-\rho)(p_s^p+p_s^g)] = b\lambda_s^{rio} - \frac{1-(1-s)^{1-\zeta}}{1-\zeta}$$
$$+\frac{\beta(1-\rho)(p_s^p+p_s^g)}{1-\beta(1-\rho)}\left[(1-\tau_s^w)\lambda_s^{rio}W_s^p h - \frac{1-(1-h-s)^{1-\zeta}}{1-\zeta} + \beta\rho\lambda_s^{rio}\right]$$
$$\Leftrightarrow \lambda_s^{S_o}\left[1-\beta+\beta(1-\rho)(p_s^p+p_s^g)\left(1-\frac{\beta\rho}{1-\beta(1-\rho)}\right)\right] = b\lambda_s^{rio} - \frac{1-(1-s)^{1-\zeta}}{1-\zeta}$$
$$+\frac{\beta(1-\rho)(p_s^p+p_s^g)}{1-\beta(1-\rho)}\left[(1-\tau_s^w)\lambda_s^{rio}W_s^p h - \frac{1-(1-h-s)^{1-\zeta}}{1-\zeta}\right]$$

$$\Leftrightarrow \lambda_s^{S_o} = \frac{b\lambda_s^{rio} - B_1^S + B_2^S W_s^p h\lambda_s^{rio}}{B_3^S} \tag{C.40}$$

with

$$B_1^S = \frac{1-(1-s)^{1-\zeta}}{1-\zeta} + \frac{\beta(1-\rho)(p_s^p+p_s^g)}{1-(1-\rho)\beta}\frac{1-(1-h-s)^{1-\zeta}}{(1-\zeta)}$$
$$B_2^S = \frac{\beta(1-\rho)(p_s^p+p_s^g)}{1-\beta(1-\rho)}(1-\tau_s^w)$$
$$B_3^S = 1-\beta+\beta(1-\rho)(p_s^p+p_s^g)\left(1-\frac{\beta\rho}{1-\beta(1-\rho)}\right)$$

For non-Ricardian workers

In a similar way, we obtain:

$$\lambda_s^{E_{rp}} = \frac{1}{1-(1-\rho)\beta}\left[(1-\tau_s^w)W_s^p h\lambda_s^{rir} - \frac{1-(1-h-s)^{1-\zeta}}{(1-\zeta)} + \beta\rho\lambda_s^{S_{rs}}\right] \tag{C.41}$$

$$\lambda_s^{E_{rg}} = \frac{1}{1-(1-\rho)\beta}\left[(1-\tau_s^w)W_s^p h\lambda_s^{rir} - \frac{1-(1-h-s)^{1-\zeta}}{(1-\zeta)} + \beta\rho\lambda_s^{S_{rs}}\right] \tag{C.42}$$

$$\lambda_s^{S_r} = \frac{b\lambda_s^{rir} - B_1^S + B_2^S W_s^p h\lambda_s^{rir}}{B_3^S} \tag{C.43}$$

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